DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.1 REAL NUMBERS

This chapter consists of four different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 EUCLID'S DIVISION LEMMA

1. Express107 in the form of 4q+3 for some positive integer q.
2. Show that any positive odd integer is of the form 6q+1 or 6q+3 or 6q+5, where q is some integer.
3. Find the HCF of 65 and 117 and express it in the form of $65m + 117n$.

TYPE: 2 HCF AND LCM

1. Find the (HCF x LCM) for the numbers 100 and 190
2. Given that LCM $(26,169) = 338$, write HCF (26,169).
3. Find the HCF and LCM of 6, 72 and 120 using the prime factorization method.
4. Using prime factorization method, find the HCF and LCM of 72,126 and 168. Also show that $HCF \times LCM \neq$ product of three numbers.
5. A person wanted to distribute 96 Apples and 112 oranges among poor children in an orphanage. He packed all the fruits in boxes in such away that each box contains fruits of the same variety, and also every box contains an equal number of fruits. (a) Find the maximum number of boxes in which all the fruits can be packed. (b) Which concept have you used to find it?

TYPE: 3 FUNDAMENTAL THEOREM OF ARITHMETIC AND IRRATIONAL NUMBERS

1. State the fundamental theorem of Arithmetic.
2. Check whether 6^n can end with the digit 0, where n is any natural number.
3. Check whether $5 \times 3 \times 11 + 22$ and $5 \times 7 + 7 \times 3$ are composite number and justify.

TYPE: 4 IRRATIONAL NUMBERS

1. Show that $5 + 3\sqrt{2}$is an irrational number.
2. Prove that $\sqrt{7}$is irrational.
3. State whether the number $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$ is rational or irrational justify.
4. Write whether the rational number $\frac{51}{1500}$will have a terminating decimal expansion or a non terminating repeating decimal expansion.

Ch.1 Real Numbers

Exercise 1.1 :

Q1 :

Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Answer :

(i) 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

225 = 135 x 1 + 90

Since remainder 90 ≠ 0, we apply the division lemma to 135 and 90 to obtain

135 = 90 x 1 + 45

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

90 = 2 x 45 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii)196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

38220 = 196 x 195 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii)867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

867 = 255 x 3 + 102

Since remainder 102 ≠ 0, we apply the division lemma to 255 and 102 to obtain

255 = 102 x 2 + 51

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

102 = 51 x 2 + 0

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

Q2 :

Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Answer :

Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$for some integer q is not 0, and = 0, 1, 2, 3, 4, 5 because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where$k_1$ is a positive integer

$6q + 3 = (6q + 2) + 1 = 2 (3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2 (3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2. Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$,

or $6q + 5$

Q3 :

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer :

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Q4 :

Use Euclid's division lemma to show that the square of any positive integer is either of form 3*m* or 3*m* + 1 for some integer *m*.

[Hint: Let *x* be any positive integer then it is of the form 3*q*, 3*q* + 1 or 3*q* + 2. Now square each of these and show that they can be rewritten in the form 3*m* or 3*m* + 1.]

Answer :

Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$\begin{aligned}
&a^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3\text{q}+2)^2 \\
&a^2 = (9q^2) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4 \\
&= 3 \times (3q^2) \text{ or } 3(3q^2 + 2q) + 1 \text{ or } 3(3\text{q}^2 + 4q + 1) + 1 \\
&= 3k_1 \quad \text{or} \quad 3k_2 + 1 \quad \text{or} \quad 3k_3 + 1
\end{aligned}$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form 3*m* or 3*m* + 1.

Q5 :

Use Euclid's division lemma to show that the cube of any positive integer is of the form 9*m*, 9*m* + 1 or 9*m* +8.

Answer :

Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$$\therefore a = 3q \text{ or } 3q + 1 \text{ or } 3q + 2$$

Therefore, every number can be represented as these three forms. There are three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m,$$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Exercise 1.2 :

Q1 :

Express each number as product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Answer :

(i) $140 = 2\times2\times5\times7 = 2^2\times5\times7$

(ii) $156 = 2\times2\times3\times13 = 2^2\times3\times13$

(iii) $3825 = 3\times3\times5\times5\times17 = 3^2\times5^2\times17$

(iv) $5005 = 5\times7\times11\times13$

(v) $7429 = 17\times19\times23$

Q2 :

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Answer :

(i) 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

$\text{HCF} = 13$

$\text{LCM} = 2 \times 7 \times 13 = 182$

Product of the two numbers $= 26 \times 91 = 2366$

$\text{HCF} \times \text{LCM} = 13 \times 182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$\text{HCF} = 2$

$\text{LCM} = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of the two numbers $= 510 \times 92 = 46920$

$$\begin{aligned}\text{HCF} \times \text{LCM} &= 2 \times 23460 \\ &= 46920\end{aligned}$$

Hence, product of two numbers = HCF × LCM

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

$\text{HCF} = 2 \times 3 = 6$

$\text{LCM} = 2^4 \times 3^3 \times 7 = 3024$

Product of the two numbers $= 336 \times 54 = 18144$

$\text{HCF} \times \text{LCM} = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

Q3 :

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Answer :

(i) 12,15 and 21

$$12 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$21 = 3 \times 7$$
$$\text{HCF} = 3$$
$$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$$

(ii) 17, 23 and 29

$$17 = 1 \times 17$$
$$23 = 1 \times 23$$
$$29 = 1 \times 29$$
$$\text{HCF} = 1$$
$$\text{LCM} = 17 \times 23 \times 29 = 11339$$

(iii) 8, 9 and 25

$$8 = 2 \times 2 \times 2$$
$$9 = 3 \times 3$$
$$25 = 5 \times 5$$
$$\text{HCF} = 1$$
$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$$

Q4 :

Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer :

$$\text{HCF}(306,\ 657) = 9$$

We know that, LCM × HCF = Product of two numbers

$$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$$
$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$
$$\text{LCM} = 22338$$

Q5 :

Check whether 6^n can end with the digit 0 for any natural number *n*.

Answer :

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as 10 = 2 x 5

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of *n*, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number *n*.

Q6 :

Explain why 7 x 11 x 13 + 13 and 7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 are composite numbers.

Answer :

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

7 x 11 x 13 + 13 = 13 x (7 x 11 + 1) = 13 x (77 + 1)

= 13 x 78

= 13 x 13 x 6

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 = 5 x (7 x 6 x 4 x 3 x 2 x 1 + 1)

= 5 x (1008 + 1)

= 5 x 1009

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Q7 :

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer :

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the

LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

18 = 2 **x** 3 **x** 3

And, 12 = 2 **x** 2 **x** 3

LCM of 12 and 18 = 2 x 2 x 3 x 3 = 36

Therefore, Ravi and Sonia will meet together at the starting pointafter 36 minutes.

Exercise 1.3 :

Q1 :

Prove that $\sqrt{5}$ is irrational.

Answer :

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that $\sqrt{5} = \frac{a}{b}$

Let *a* and *b* have a common factor other than 1. Then we can divide them by the common factor, and assume that *a* and *b*are co-prime.

$$a = \sqrt{5}b$$
$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that *a* is divisible by 5.

Let *a* = 5*k*, where *k* is an integer

$$(5k)^2 = 5b^2$$
$$b^2 = 5k^2$$

This means that b^2 is divisible by 5 and hence, *b* is divisible by 5.

This implies that *a* and *b* have 5 as a common factor.

And this is a contradiction to the fact that *a* and *b* are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Q2 :

Prove that $3+2\sqrt{5}$ is irrational.

Answer :

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$$3+2\sqrt{5}=\frac{a}{b}$$

$$2\sqrt{5}=\frac{a}{b}-3$$

$$\sqrt{5}=\frac{1}{2}\left(\frac{a}{b}-3\right)$$

Since *a* and *b* are integers, $\frac{1}{2}\left(\frac{a}{b}-3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false.

Therefore, $3+2\sqrt{5}$ is irrational.

Q3 :

Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$

Answer :

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$$\frac{1}{\sqrt{2}}=\frac{a}{b}$$

$$\sqrt{2}=\frac{b}{a}$$

$\frac{b}{a}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

$(\text{ii}) \quad 7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$7\sqrt{5} = \frac{a}{b}$ for some integers *a* and *b*

$$\therefore \sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

$(\text{iii}) \quad 6+\sqrt{2}$

Let $6+\sqrt{2}$ be rational.

Therefore, we can find two integers *a*, *b* (*b* ≠ 0) such that

$$6+\sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

Since *a* and *b* are integers, $\frac{a}{b} - 6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false and hence,

Exercise 1.4 :

Q1 :

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Answer :

(i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

455 = 5 × 7 × 13

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$1600 = 2^6 \times 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 \times 5^2}$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50} = \frac{7 \times 5}{10 \times 5} = \frac{7}{10}$

$10 = 2 \times 5$

The denominator is of the form $2^m \times 5^n$.

Q2 :

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer :

(i) $\frac{13}{3125} = 0.00416$

$$
\begin{array}{r}
0.00416 \\
3125\overline{)13.00000} \\
\underline{0} \\
130 \\
\underline{0} \\
1300 \\
\underline{0} \\
13000 \\
\underline{12500} \\
5000 \\
\underline{3125} \\
18750 \\
\underline{18750} \\
\underline{\times}
\end{array}
$$

(ii) $\frac{17}{8} = 2.125$

$$
\begin{array}{r}
2.125 \\
8\overline{)\;17} \\
\underline{16} \\
10 \\
\underline{8} \\
20 \\
\underline{16} \\
40 \\
\underline{40} \\
\underline{\times}
\end{array}
$$

(iv) $\frac{15}{1600} = 0.009375$

$$\begin{array}{r} 0.009375 \\ 1600\overline{)15.000000} \\ 0 \\ \hline 150 \\ 0 \\ \hline 1500 \\ 0 \\ \hline 15000 \\ 14400 \\ \hline 6000 \\ 4800 \\ \hline 12000 \\ 11200 \\ \hline 8000 \\ 8000 \\ \hline \times \\ \hline \end{array}$$

(vi) $\frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$

$$\begin{array}{r} 0.115 \\ 200\overline{)23.000} \\ 0 \\ \hline 230 \\ 200 \\ \hline 300 \\ 200 \\ \hline 1000 \\ 1000 \\ \hline \times \\ \hline \end{array}$$

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ 0 \\ \hline 20 \\ 20 \\ \hline \times \\ \hline \end{array}$$

(ix) $\frac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 0 \\ \hline 350 \\ 350 \\ \hline \times \\ \hline \end{array}$$

Q3 :

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of *q*?

(i) 43.123456789 (ii) 0.120120012000120000… (iii) $43.\overline{123456789}$

Answer :

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and *q* is of the form $2^m \times 5^n$

i.e., the prime factors of *q* will be either 2 or 5 or both.

(ii) 0.120120012000120000 …

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

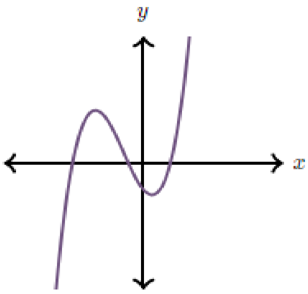
Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and *q* is not of the form $2^m \times 5^n$ i.e., the prime factors of *q* will also have a factor other than 2 or 5.

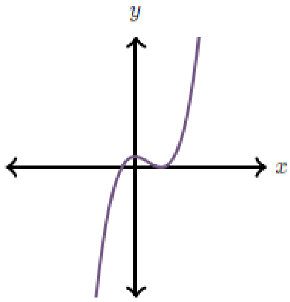
DELHI PUBLIC SCHOOL, GANDHINAGAR
CH. 2 POLYNOMIALS
MIND MAP

This chapter consists of three different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 GEOMETRICAL MEANING OF ZEROES OF A POLYNOMIAL

1. Find the number of zeroes from the graph:





TYPE: 2 ZEROES OF A QUADRATIC POLYNOMIAL

1. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$?
2. If α and β are the zeros of quadratic polynomial $p(x) = x^2 - (k-6)x + (2k+1)$. Find the value of k if $\alpha + \beta = \alpha\beta$.
3. If the sum of squares of zeros of the polynomial $6x^2 + x + k$ is $\frac{25}{36}$ find the value of k.
4. Find the quadratic polynomial sum of whose zeros is $2\sqrt{3}$ and their product is 2.
5. If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx - 9$ is negative of the other, find the value of k.

TYPE: 3 DIVISION ALGORITHM OF POLYNOMIALS

1. What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by$x^2 - 4x + 3$.
2. Can $x - 7$ be the remainder on division of a polynomial $P(x)$by $7x + 2$? Justify your answer..
3. On dividing $3x^3 - 2x^2 + 5x - 5$ by the polynomial p(x), the quotient and remainder are $x^2 - x + 2$ and -7 respectively. Find $p(x)$.

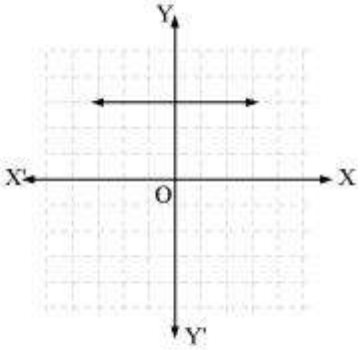
Ch.2 POLYNOMIALS

Exercise 2.1

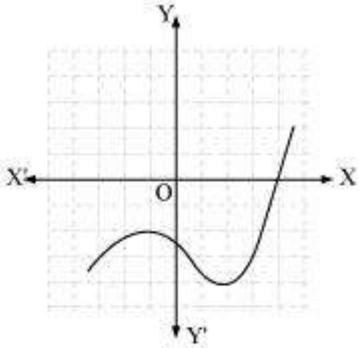
Q1 :

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of$p(x)$, in each case.

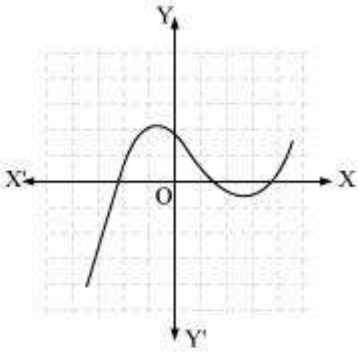
(i)



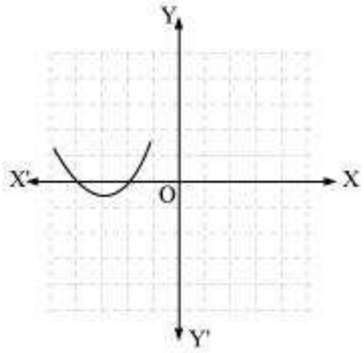
(ii)



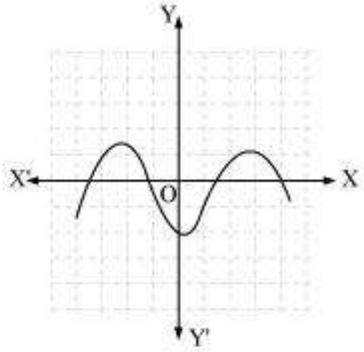
(iii)



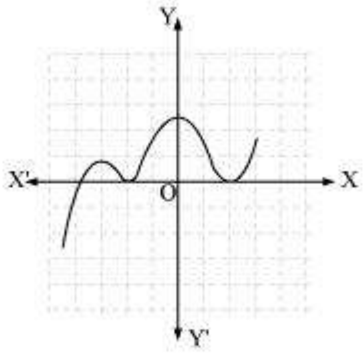
(iv)



(v)



(v)



Answer :

(i) The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the *x*-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the *x*-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the *x*-axis at 3 points.

Exercise 2.2

Q1 :

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2 - 2x - 8 \quad (\text{ii})\, 4s^2 - 4s + 1 \quad (\text{iii})\, 6x^2 - 3 - 7x$

$(\text{iv})\, 4u^2 + 8u \quad (\text{v})\, t^2 - 15 \quad (\text{vi})\, 3x^2 - x - 4$

Answer :

$(\text{i}) \qquad x^2 - 2x - 8 = (x-4)(x+2)$

The value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e., when $x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and - 2.

Sum of zeroes = $4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $= 4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$(\text{ii}) \qquad 4s^2 - 4s + 1 = (2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

$(\text{iii}) \qquad 6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$\begin{aligned}(\text{iv})\quad & 4u^2+8u=4u^2+8u+0\\ & =4u(u+2)\end{aligned}$$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = - 2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and - 2.

Sum of zeroes = $0+(-2)=-2=\frac{-(8)}{4}=\frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

$$\begin{aligned}(\text{v})\quad & t^2-15\\ & =t^2-0t-15\\ & =\left(t-\sqrt{15}\right)\left(t+\sqrt{15}\right)\end{aligned}$$

The value of t^2 - 15 is zero when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when

Q2 :

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) $0,\sqrt{5}$

(iv) $1,1$ (v) $-\frac{1}{4},\frac{1}{4}$ (vi) $4,1$

Answer :

(i) $\frac{1}{4},-1$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$$

$$\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$$

If $a=4$, then $b=-1$, $c=-4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\sqrt{2}=\frac{3\sqrt{2}}{3}=\frac{-b}{a}$$

$$\alpha\beta=\frac{1}{3}=\frac{c}{a}$$

If $a=3$, then $b=-3\sqrt{2}$, $c=1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=0=\frac{0}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=\sqrt{5}=\frac{\sqrt{5}}{1}=\frac{c}{a}$$

If $a=1$, then $b=0$, $c=\sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$

If $a=1$, then $b=-1$, $c=1$

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be ax^2+bx+c , and its zeroes be α and

Exercise 2.3

Q1 :

Divide the polynomial *p*(*x*) by the polynomial *g*(*x*) and find the quotient and remainder in each of the following:

(i) $p(x)=x^3-3x^2+5x-3, \qquad g(x)=x^2-2$

(ii) $p(x)=x^4-3x^2+4x+5, \qquad g(x)=x^2+1-x$

(iii) $p(x)=x^4-5x+6, \qquad g(x)=2-x^2$

Answer :

$$\begin{aligned} &(\text{i}) \quad p(x)=x^3-3x^2+5x-3 \\ &\qquad\ \ q(x)=x^2-2 \end{aligned}$$

$$\begin{array}{r|l} & x-3 \\ \hline x^2-2 & x^3-3x^2+5x-3 \\ & x^3 \qquad -2x \\ & - \qquad\ \ + \\ \hline & -3x^2+7x-3 \\ & -3x^2 \qquad +6 \\ & + \qquad\quad - \\ \hline & \qquad\quad 7x-9 \\ \hline \end{array}$$

Quotient = x - 3

Remainder = 7x - 9

$$\begin{aligned} &(\text{ii}) \quad p(x)=x^4-3x^2+4x+5=x^4+0.x^3-3x^2+4x+5 \\ &\qquad\ \ \ q(x)=x^2+1-x=x^2-x+1 \end{aligned}$$

$$\begin{array}{r|l}
 & x^2+x-3 \\
\cline{2-2}
x^2-x+1 & x^4+0.x^3-3x^2+4x+5 \\
 & x^4-\ x^3+x^2 \\
 & -\quad +\quad - \\
\cline{2-2}
 & \qquad x^3-4x^2+4x+5 \\
 & \qquad x^3-\ x^2+\ x \\
 & \qquad -\quad +\quad - \\
\cline{2-2}
 & \qquad\quad -3x^2+3x+5 \\
 & \qquad\quad -3x^2+3x-3 \\
 & \qquad\quad +\quad -\quad + \\
\cline{2-2}
 & \qquad\qquad\qquad 8 \\
\cline{2-2}
\end{array}$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$\begin{array}{r|l}
 & -x^2-2 \\
\cline{2-2}
-x^2+2 & x^4+0.x^2-5x+6 \\
 & x^4-2x^2 \\
 & -\quad + \\
\cline{2-2}
 & \quad 2x^2-5x+6 \\
 & \quad 2x^2\qquad -4 \\
 & \quad -\qquad\quad + \\
\cline{2-2}
 & \qquad -5x+10 \\
\cline{2-2}
\end{array}$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Q2 :

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$

Answer :

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$
$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$
$$= 0$$

$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$
$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$
$$= -16 + 4 + 10 + 2 = 0$$

Therefore, $\frac{1}{2}$, 1, and - 2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain *a* = 2, *b* = 1, *c* = - 5, *d* = 2

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2 = 0$$

$$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$$
$$= 1 - 4 + 5 - 2 = 0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.
Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1) =2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Q3 :

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer :

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$t^2 - 3 = t^2 + 0.t - 3$

$$\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\ \hline
t^2 + 0.t - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 + 0.t^3 - 6t^2 \\
 & - \quad - \quad + \\ \cline{2-2}
 & \quad 3t^3 + 4t^2 - 9t - 12 \\
 & \quad 3t^3 + 0.t^2 - 9t \\
 & \quad - \quad - \quad + \\ \cline{2-2}
 & \qquad 4t^2 + 0.t - 12 \\
 & \qquad 4t^2 + 0.t - 12 \\
 & \qquad - \quad - \quad + \\ \cline{2-2}
 & \qquad\qquad 0 \\ \hline\hline
\end{array}$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1,\ \ 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad\ - \quad\ - \\
\cline{2-2}
 & \quad -4x^3 - 10x^2 + 2x + 2 \\
 & \quad -4x^3 - 12x^2 - 4x \\
 & \quad + \quad\ + \quad\ + \\
\cline{2-2}
 & \qquad\qquad 2x^2 + 6x + 2 \\
 & \qquad\qquad 2x^2 + 6x + 2 \\
\cline{2-2}
 & \qquad\qquad\qquad 0 \\
\cline{2-2}
\end{array}$$

Since the remainder is 0,

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1,\ \ x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r|l}
 & \quad x^2 - 1 \\
\hline
x^3 - 3x + 1 & x^5 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 - 3x^3 + x^2 \\
 & - \quad + \quad - \\
\cline{2-2}
 & \quad -x^3 \qquad + 3x + 1 \\
 & \quad -x^3 \qquad + 3x - 1 \\
 & \quad + \qquad\quad - \quad + \\
\cline{2-2}
 & \qquad\qquad\qquad 2 \\
\cline{2-2}
\end{array}$$

Since the remainder $\neq 0$,

Hence, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Q4 :

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer :

Let the polynomial be ax^3+bx^2+cx+d and the zeroes be $\alpha, \beta,$ and γ.
It is given that

$$\alpha+\beta+\gamma=\frac{2}{1}=\frac{-b}{a}$$
$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{-7}{1}=\frac{c}{a}$$
$$\alpha\beta\gamma=\frac{-14}{1}=\frac{-d}{a}$$

If a = 1, then b = - 2, c = - 7, d = 14

Hence, the polynomial is $x^3-2x^2-7x+14$.

Q5 :

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer :

$p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
\hline
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\
\cline{2-2}
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3+0x^2-10x \\
 & \qquad -\quad -\quad + \\
\cline{2-2}
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad -\quad -\quad + \\
\cline{2-2}
 & \qquad\qquad\quad 0 \\
\cline{2-2}
\end{array}
$$

$$
\begin{aligned}
3x^4+6x^3-2x^2-10x-5 &= \left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right) \\
&= 3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)
\end{aligned}
$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = - 1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = - 1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$, - 1 and - 1.

Q6 :

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $- 2x + 4$, respectively. Find $g(x)$.

Answer :

$p(x)=x^3-3x^2+x+2 \qquad \text{(Dividend)}$

$g(x)$ = ? (Divisor)

Quotient = $(x - 2)$

Remainder = (- 2x + 4)

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x-2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x-2)$$

$g(x)$ is the quotient when we divide $\left(x^3 - 3x^2 + 3x - 2\right)$ by $(x-2)$

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\ \hline
x-2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\ \hline
 & \quad -x^2 + 3x - 2 \\
 & \quad -x^2 + 2x \\
 & \quad + \quad - \\ \hline
 & \qquad\quad x - 2 \\
 & \qquad\quad x - 2 \\
 & \qquad\quad - \quad + \\ \hline
 & \qquad\qquad 0 \\ \hline
\end{array}
$$

$$\therefore g(x) = \left(x^2 - x + 1\right)$$

Q7 :

Give examples of polynomial *p*(*x*), *g*(*x*), *q*(*x*) and *r*(*x*), which satisfy the division algorithm and

(i) deg *p*(*x*) = deg *q*(*x*)

(ii) deg *q*(*x*) = deg *r*(*x*)

(iii) deg *r*(*x*) = 0

Answer :

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

Here, $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$

$q(x) = 3x^2 + x + 1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = 2(3x^2 + x + 1)$

$= 6x^2 + 2x + 2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Exercise 2.4

Q1 :

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find *a* and *b*.

Answer :

$p(x) = x^3 - 3x^2 + x + 1$

Zeroes are $a - b$, $a + a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$p = 1$, $q = -3$, $r = 1$, $t = 1$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are $1 - b,\ 1,\ 1 + b$.

$$\text{Multiplication of zeroes} = 1(1 - b)(1 + b)$$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Q2 :

]It two zeroes of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$, find other zeroes.

Answer :

Given that 2 + $\sqrt{3}$ and 2 $-\sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right)$ = x^2 + 4 - 4x - 3

= x^2 - 4x + 1 is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4-6x^3-26x^2+138x-35$ byx^2 - 4x + 1.

$$\begin{array}{rl}
 & x^2-2x-35 \\ \hline
x^2-4x+1\,\big) & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+\quad x^2 \\
 & -\quad+\quad- \\ \hline
 & -2x^3-27x^2+138x-35 \\
 & -2x^3+8x^2-\quad 2x \\
 & +\quad-\quad+ \\ \hline
 & -35x^2+140x-35 \\
 & -35x^2+140x-35 \\
 & +\quad-\quad+ \\ \hline
 & 0
\end{array}$$

Clearly, $x^4-6x^3-26x^2+138x-35$ = $\left(x^2-4x+1\right)\left(x^2-2x-35\right)$

It can be observed that $\left(x^2-2x-35\right)$ is also a factor of the given polynomial.

And $\left(x^2-2x-35\right)$ = $(x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x-7=0$ or $x+5=0$

Or x = 7 or - 5

Hence, 7 and - 5 are also zeroes of this polynomial.

Q3 :

If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial x^2-2x+k, the remainder comes out to be *x* + *a*, find *k* and *a*.

Answer :

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend - Remainder = Divisor × Quotient

$x^4-6x^3+16x^2-25x+10-x-a=x^4-6x^3+16x^2-26x+10-a$ will be perfectly divisible by x^2-2x+k.

Let us divide $x^4-6x^3+16x^2-26x+10-a$ by x^2-2x+k

$$
\begin{array}{r|l}
 & x^2-4x+\left(8-k\right) \\
\hline
x^2-2x+k & x^4-6x^3+16x^2-26x+10-a \\
 & x^4-2x^3+\ kx^2 \\
 & -\quad +\qquad - \\
\hline
 & -4x^3+\left(16-k\right)x^2-26x \\
 & -4x^3+\qquad 8x^2-\ 4kx \\
 & +\qquad -\qquad\quad + \\
\hline
 & \left(8-k\right)x^2-\left(26-4k\right)x+10-a \\
 & \left(8-k\right)x^2-\left(16-2k\right)x+\left(8k-k^2\right) \\
 & -\qquad\quad +\qquad\quad - \\
\hline
 & \left(-10+2k\right)x+\left(10-a-8k+k^2\right)
\end{array}
$$

It can be observed that $\left(-10+2k\right)x+\left(10-a-8k+k^2\right)$ will be 0.

Therefore, $\left(-10+2k\right)$ = 0 and $\left(10-a-8k+k^2\right)$ = 0

For $\left(-10+2k\right)$ = 0,

2 *k* =10

And thus, *k* = 5

For $\left(10-a-8k+k^2\right)$ = 0

10 - *a* - 8 × 5 + 25 = 0

10 - *a* - 40 + 25 = 0

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$

POLYNOMIALS

POLYNOMIALS

Objective: To study

- Polynomials of degree 1, 2 &3
- The zeroes of polynomial p(x)
- Relationship between zeroes and coefficients of a polynomial
- The division algorithm of a polynomial

Do we remember this???

VARIABLES

(one, two, three, many)

DEGREE

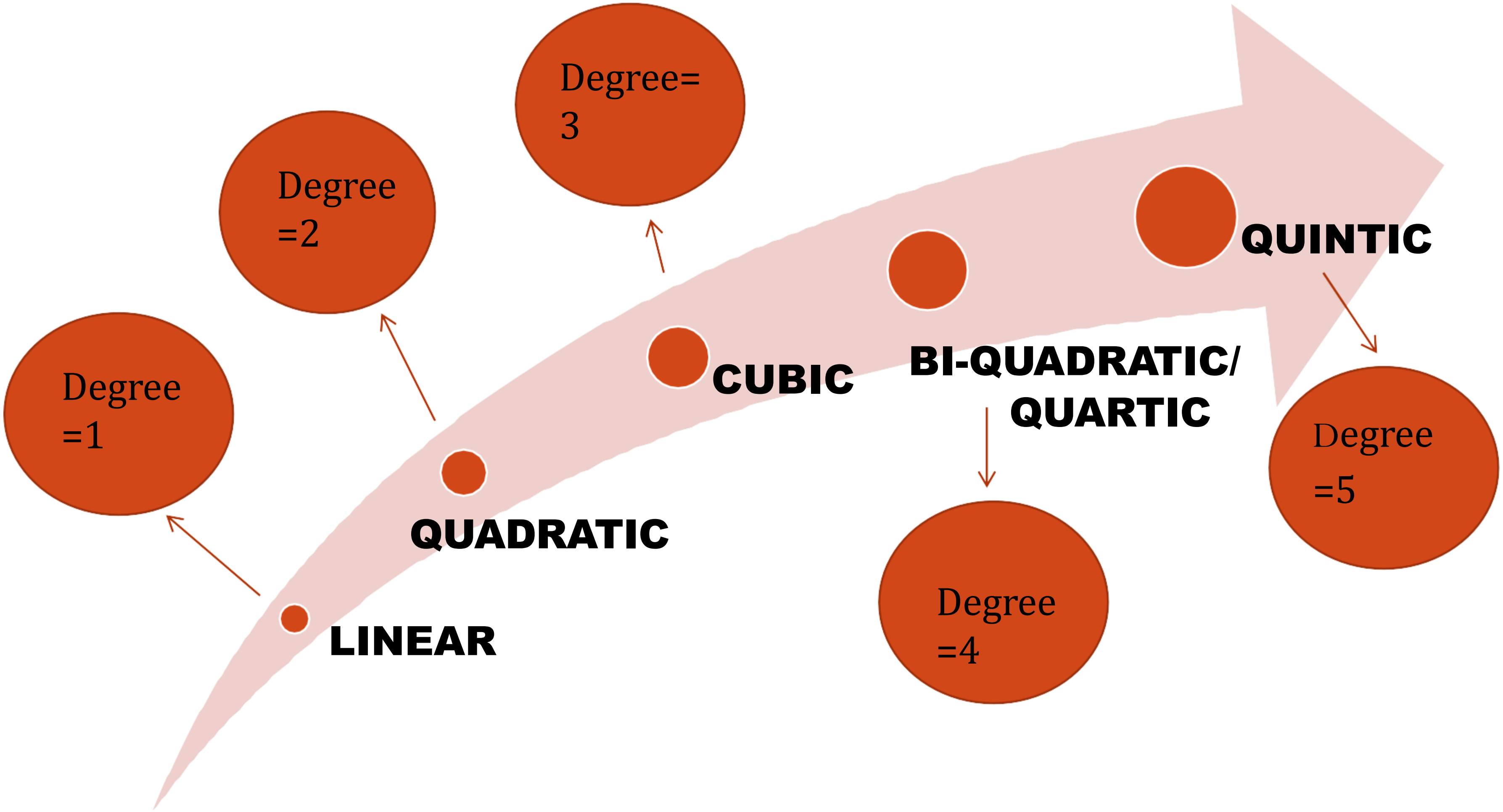
(1, 2, 3, 4, 5)

POLYNOMIAL

VALUE OF A POLYNOMIAL

ZERO OF A POLYNOMIAL

DEGREE OF A POLYNOMIAL



LINEAR

Degree
=1

QUADRATIC

Degree
=2

CUBIC

Degree=
3

BI-QUADRATIC/
QUARTIC

Degree
=4

QUINTIC

Degree
=5

VALUE OF A POLYNOMIAL

If p(x) is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in p(x) is called the **value of p(x) at x=k.**

Example: $p(x) = x^2 + 2$

Then, $p(-4) = (-4)^2 + 2$

$= 16 + 2$

$= 18$

ZERO OF A POLYNOMIAL

- A real number k, is said to be a zero of the polynomial p(x), if p(k)=0

- **Example:** For a linear polynomial p(x)= 2x+3

  Then , p(k)= 0 gives us

  2k+3 = 0. i.e. k= – 3/2

- In general, if k is a zero of p(x)= ax+b, then,

  k= – b/a

WHAT'S NEW THIS TIME?

- Geometrical meaning of the zeroes of a polynomial
- Relationship between the zeroes and coefficients of a polynomial
- Division Algorithm for Polynomials

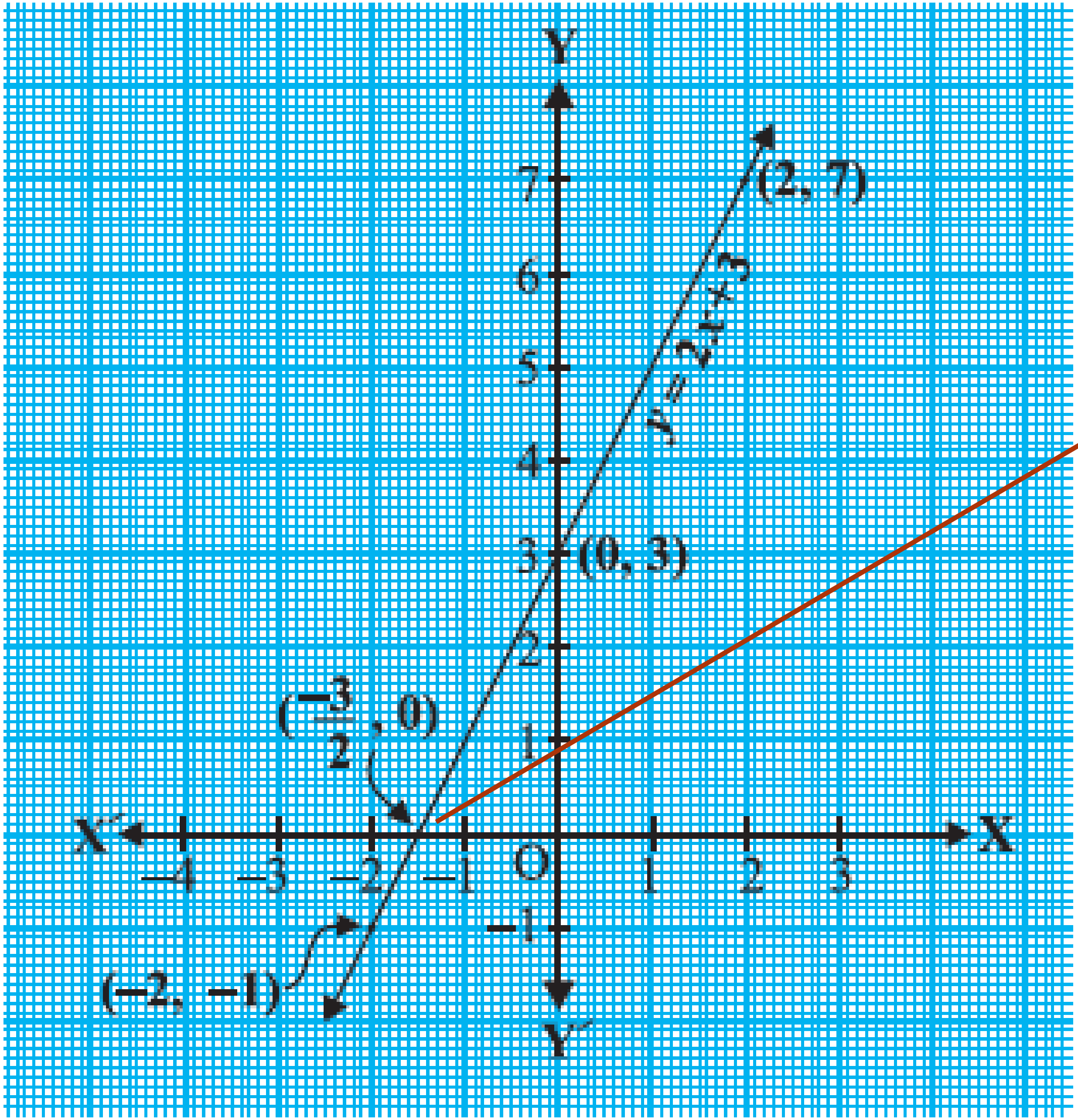
Geometrical meaning of the zeroes of a polynomial

LINEAR POLYNOMIAL

Plot the graph of the polynomial y= 2x+3

If $y = 0$,

then, $x = -\frac{3}{2}$



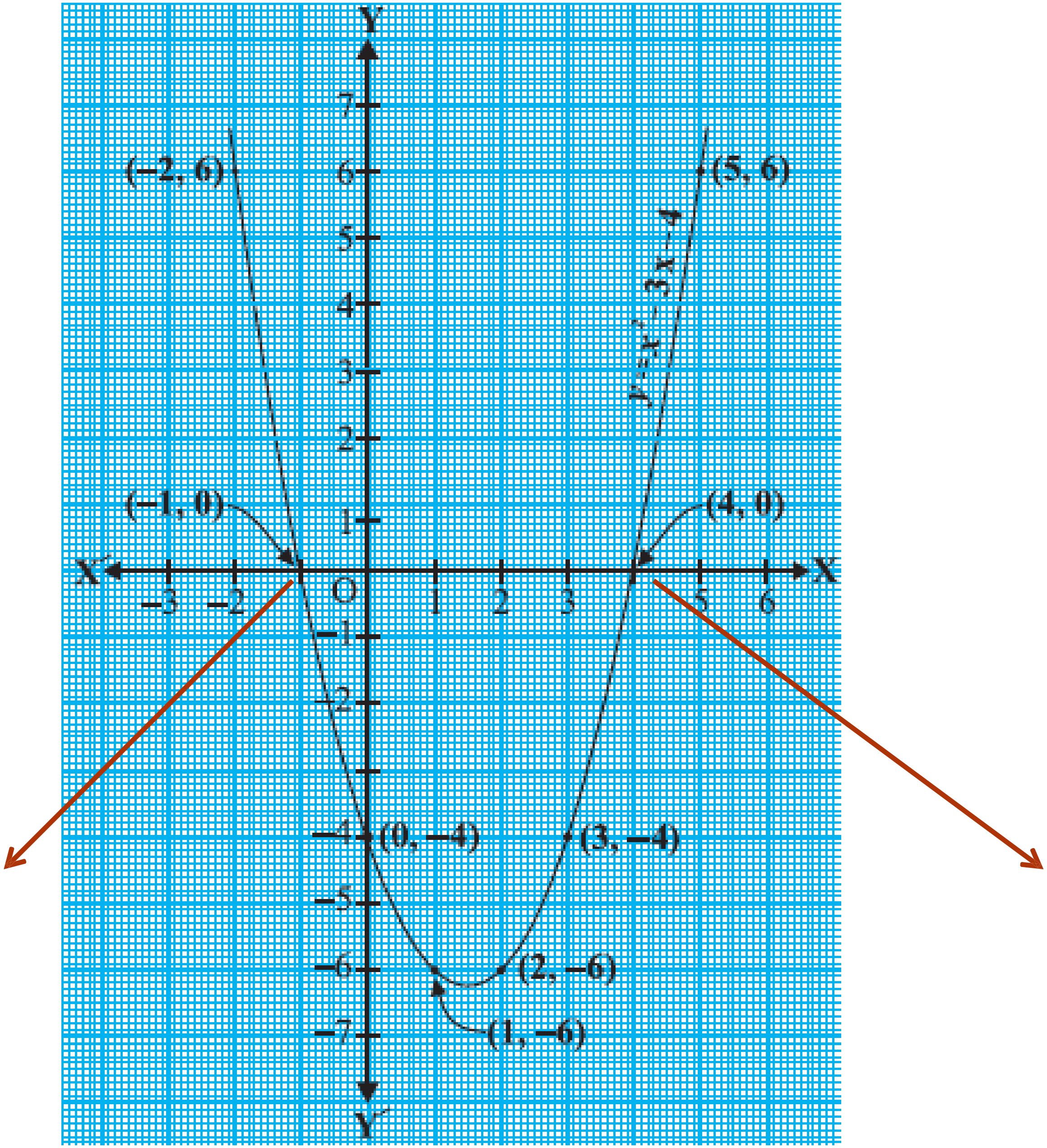
ZERO

Geometrical meaning of the zeroes of a polynomial

QUADRATIC POLYNOMIAL

Plot the graph of the polynomial $y = x^2 - 3x - 4$

$$x^2 - 3x - 4 = 0$$
$$x^2 - 4x + x - 4 = 0$$
$$(x - 4)(x + 1) = 0$$
$$(x - 4) = 0; \quad (x + 1) = 0$$
$$x = 4; \quad x = -1$$



FIRST ZERO

SECOND ZERO

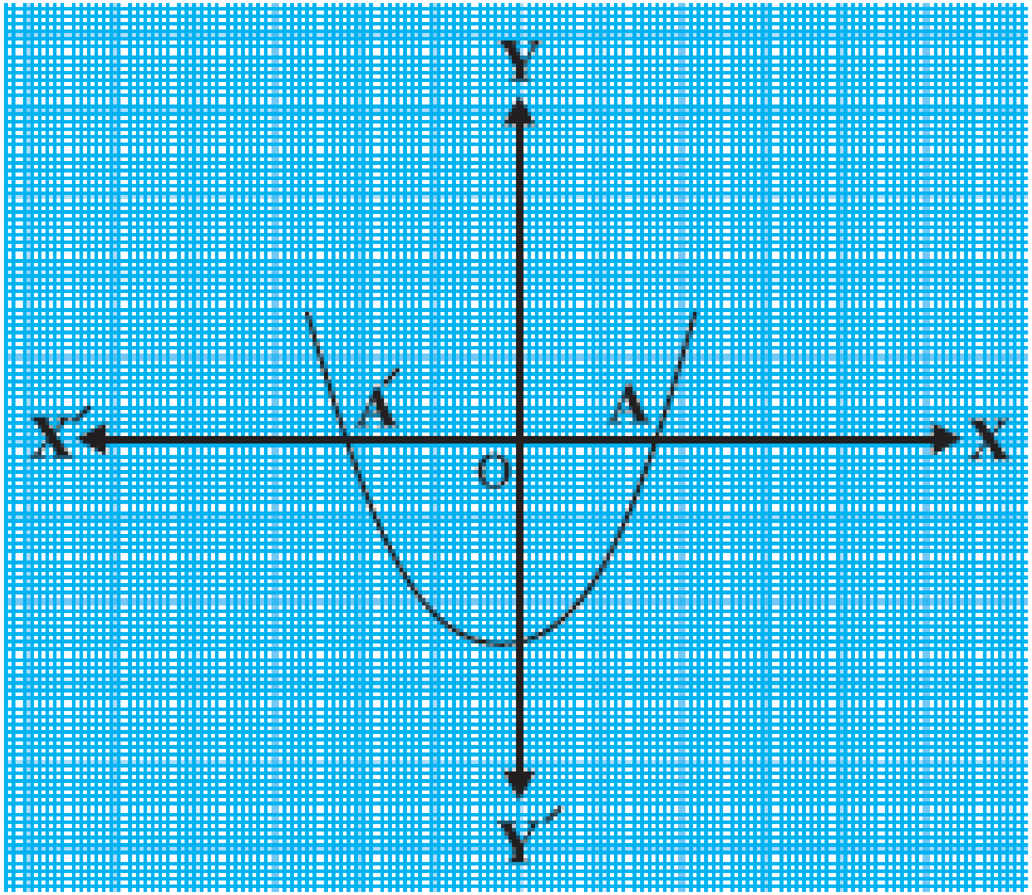
Special cases in quadratic polynomials

Question:

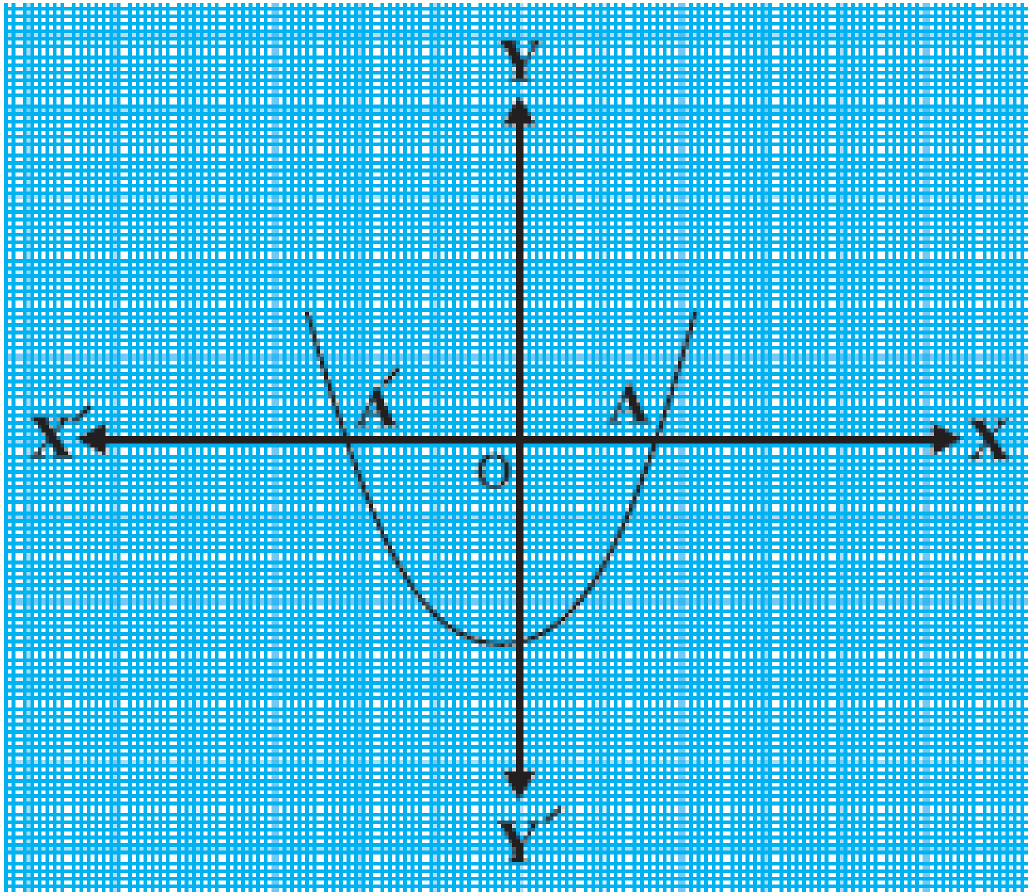
How many zeroes do the following polynomials have?

How many zeroes do the following polynomials have ?

(a)



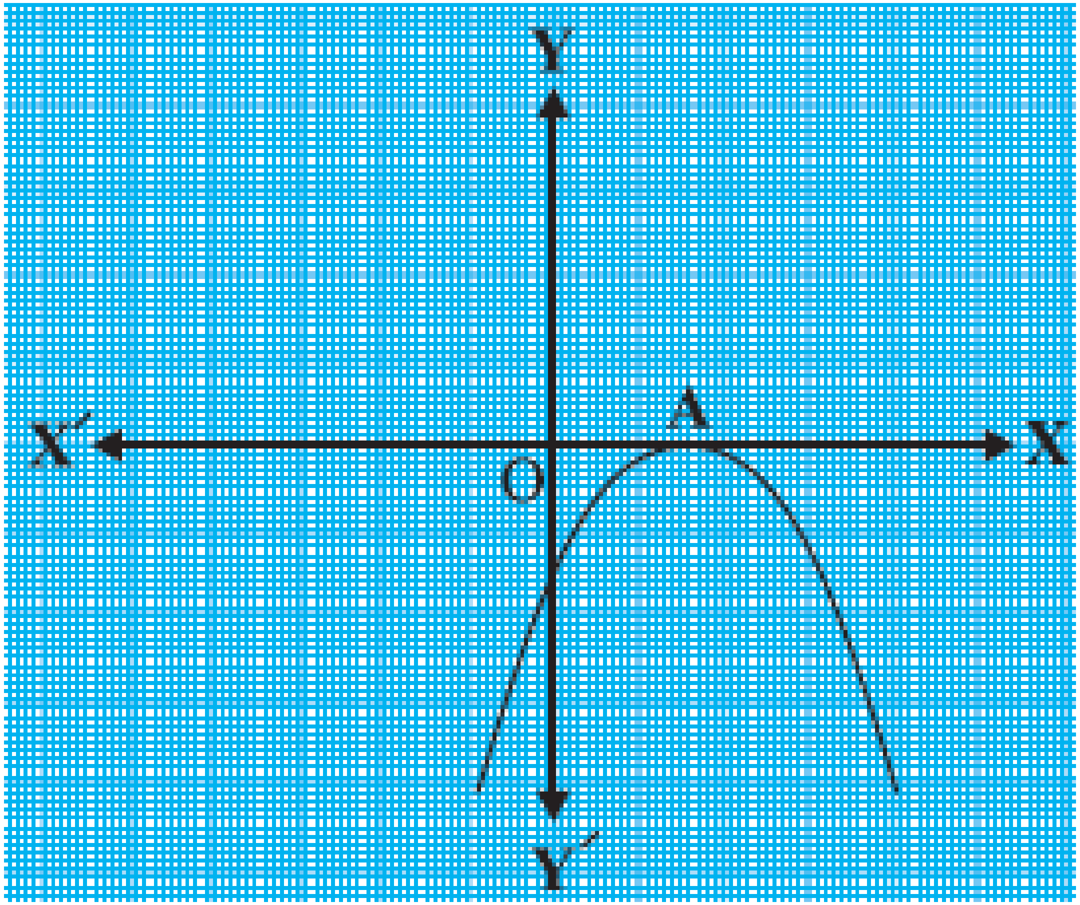
(a)



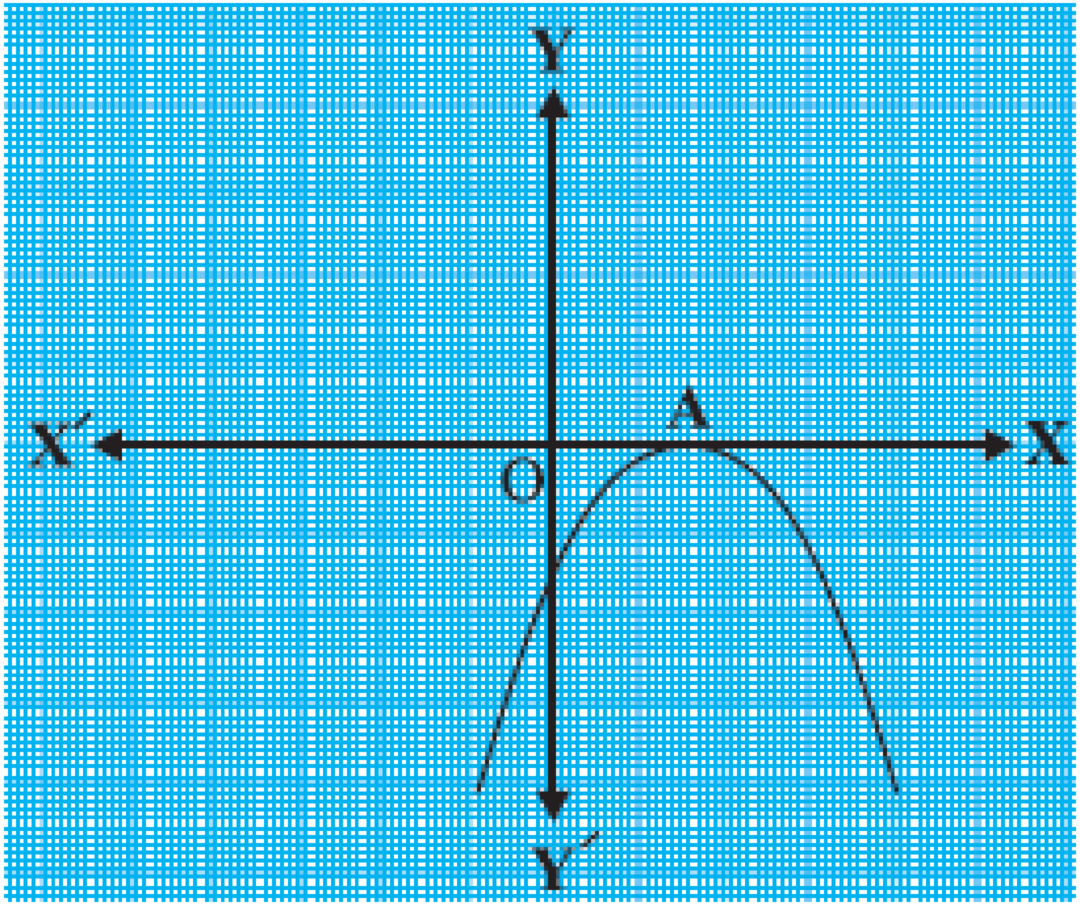
Answer: 2

How many zeroes do the following polynomials have ?

(b)



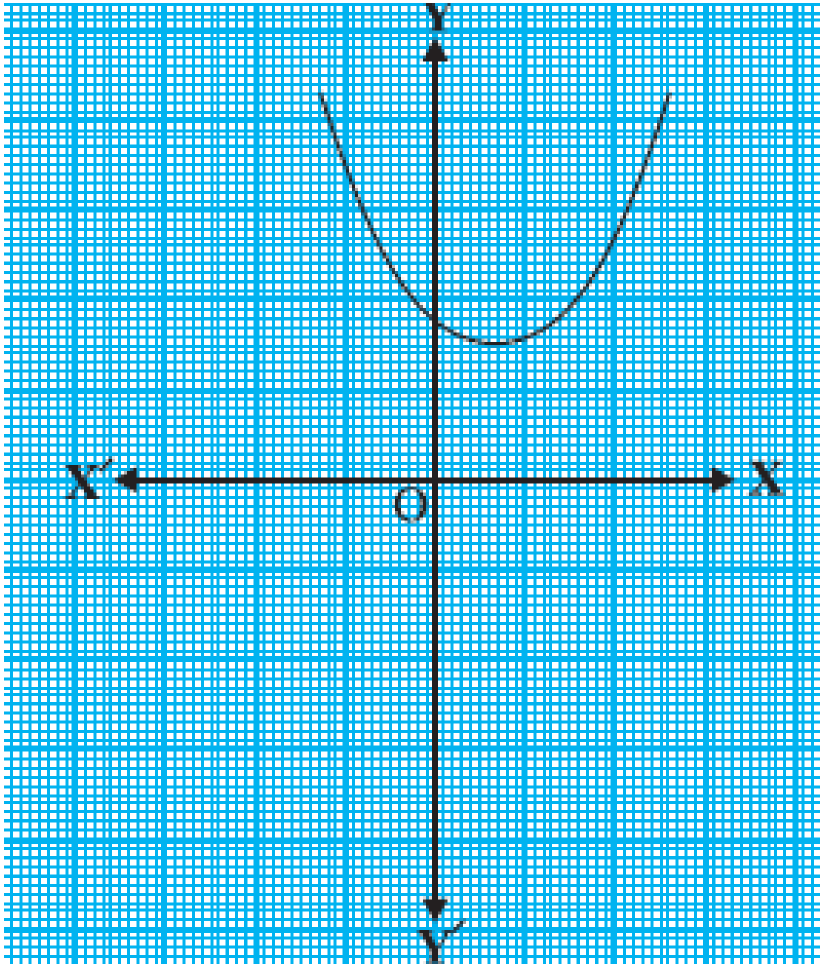
(b)



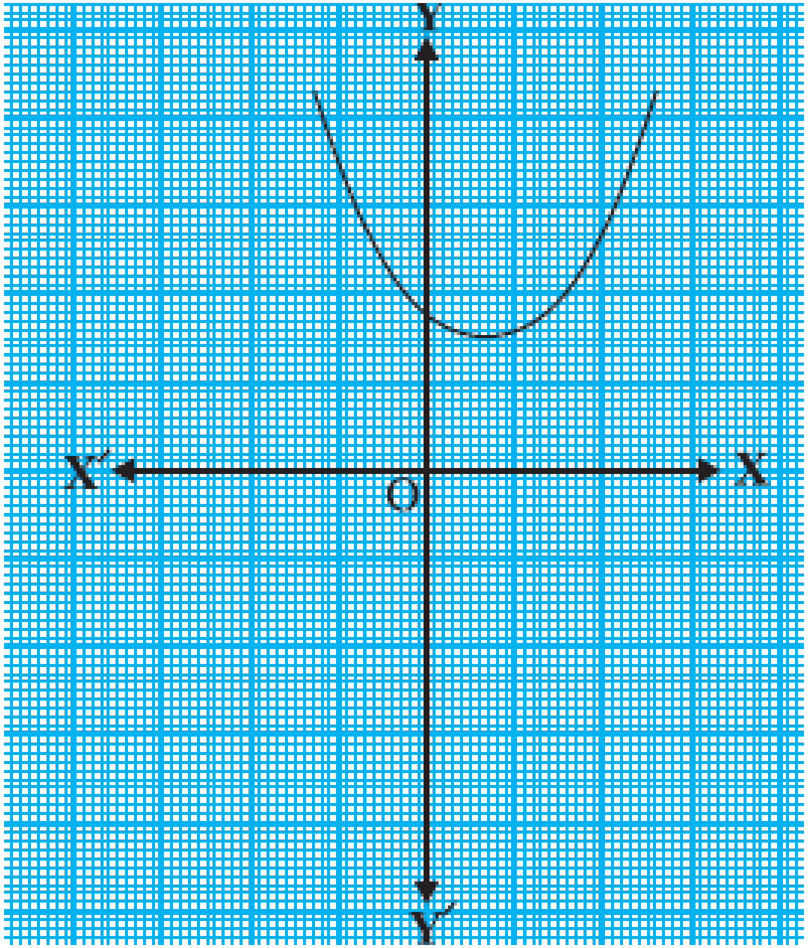
Answer: 1

How many zeroes do the following polynomials have ?

(c)



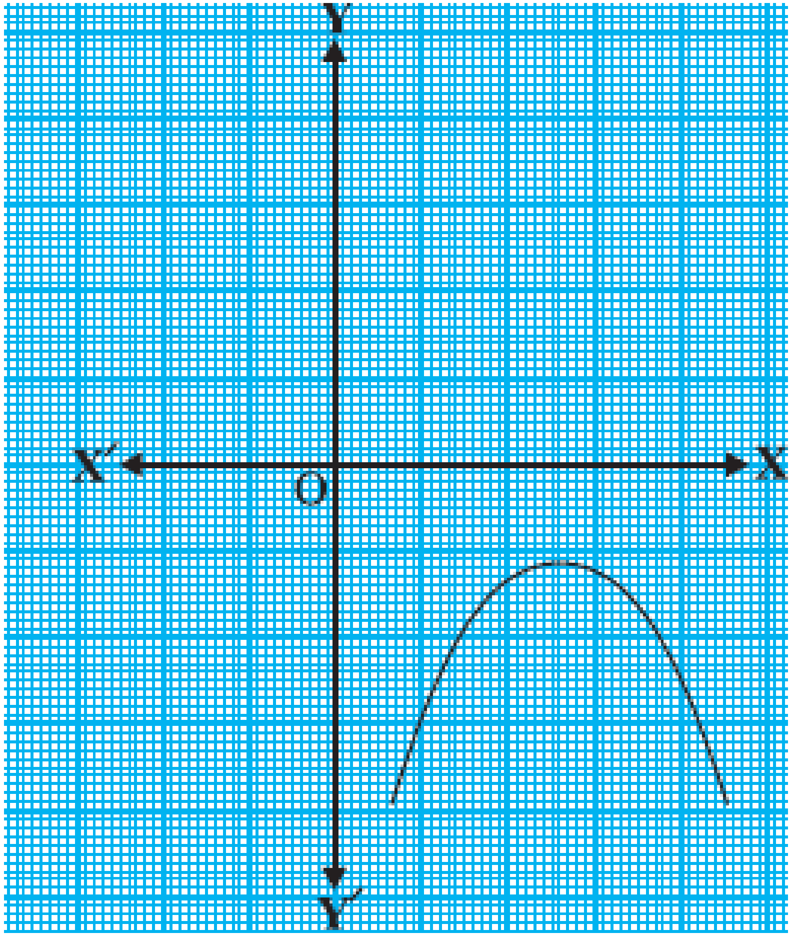
(c)



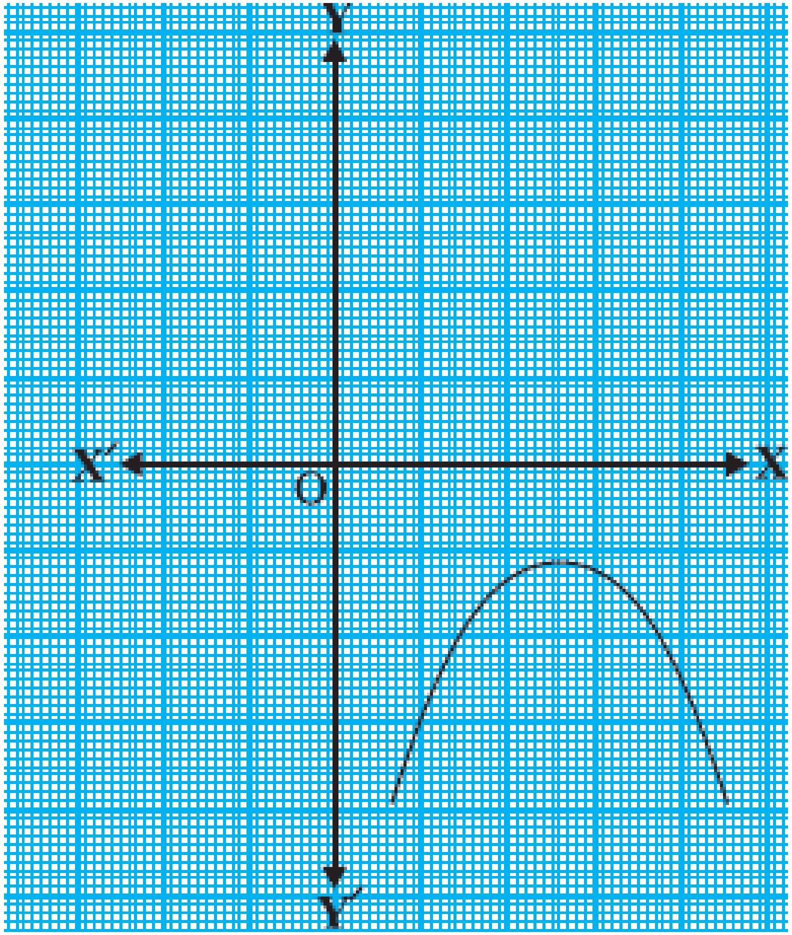
Answer: no zero

How many zeroes do the following polynomials have ?

(d)

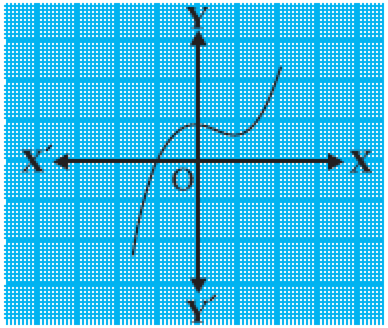


(d)

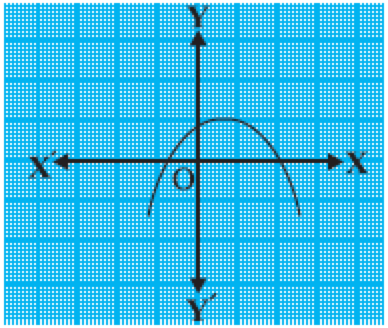


Answer: no zero

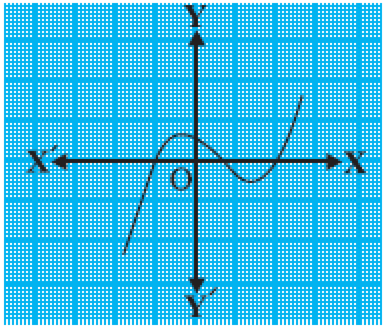
Some more............



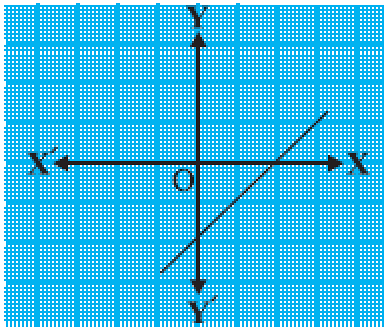
(i)



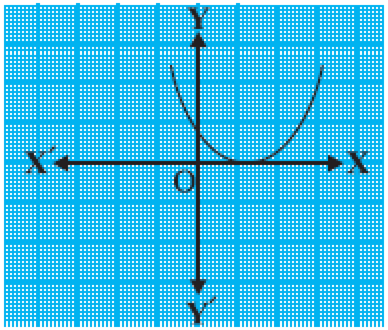
(ii)



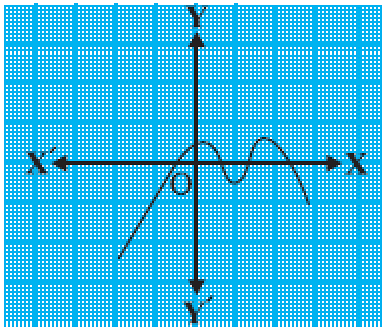
(iii)



(iv)



(v)



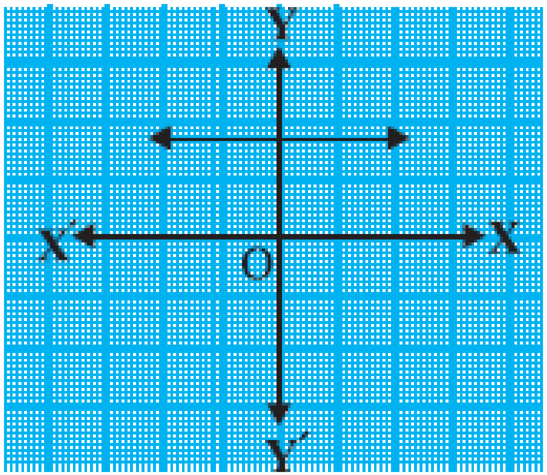
(vi)

TESTING TIME

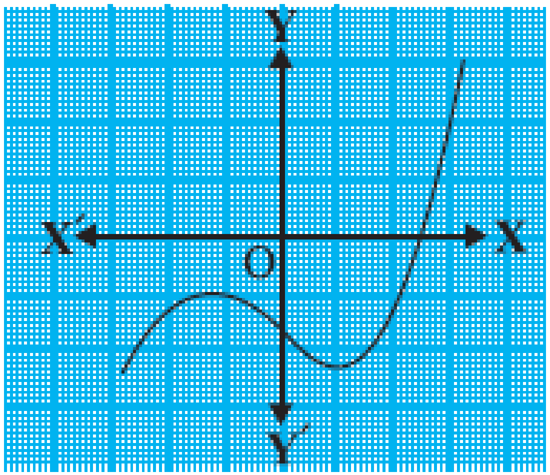
Solve Ex 2.1 and see if you can get the answers.

EXERCISE 2.1

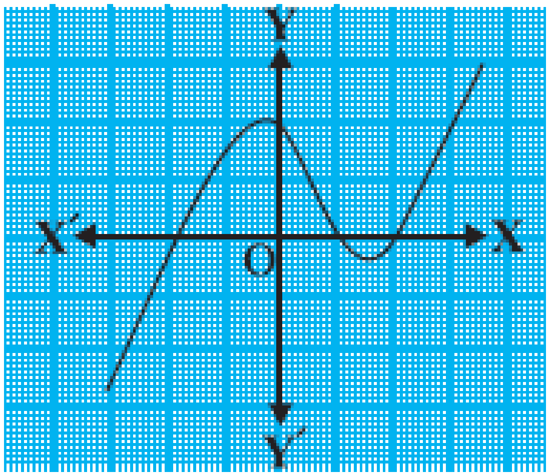
1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



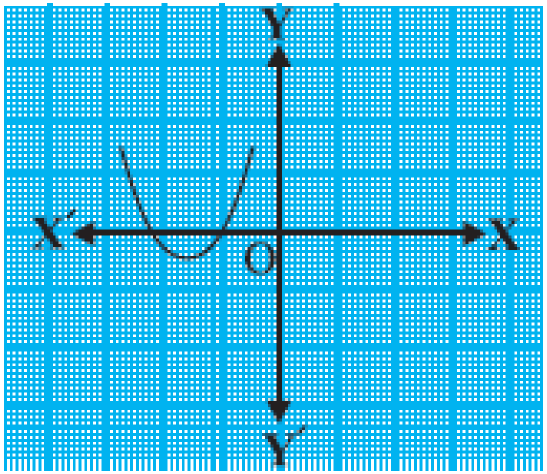
(i)



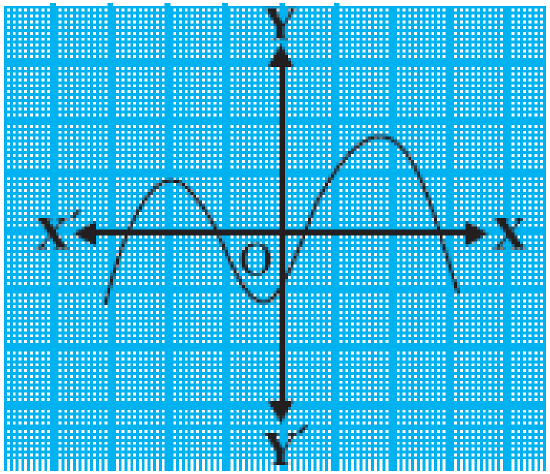
(ii)



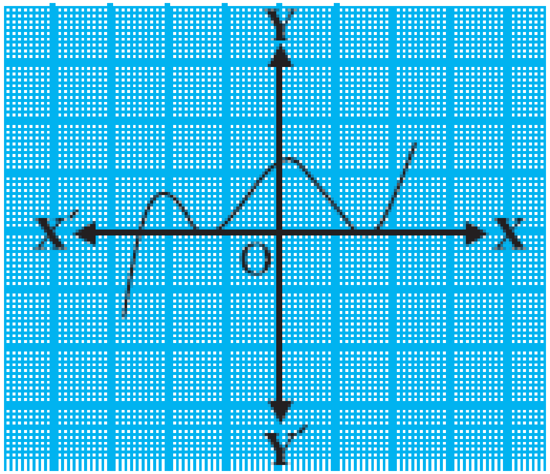
(iii)



(iv)



(v)



(vi)

WHAT'S NEW THIS TIME?

- Geometrical meaning of the zeroes of a polynomial
- Relationship between the zeroes and coefficients of a polynomial
- Division Algorithm for Polynomials

Objectives

- To find sum/product of roots *without* knowing the actual value of x.

- Use the sum/product of roots to solve for other results

Derivation of Formula

Given α and β are roots of a quadratic equation, we can infer that

$x_1 = \alpha$ and $x_2 = \beta$, where α and β are just some numbers.

For example, $x^2 - 5x + 6 = 0$ has roots α and β means

$$x^2 - 5x + 6 = 0$$
$$\therefore x^2 - 2x - 3x + 6 = 0$$
$$\therefore x(x - 2) - 3(x - 2) = 0$$
$$\therefore (x - 2)(x - 3) = 0$$
$$\therefore x = 2, \qquad x = 3$$

$\alpha = 2$ and $\beta = 3$

However...

Sometimes, we do not need the values of x to help us solve the problem.

Knowing the relationship between the quadratic equation and its roots helps us save time

Derivation of Formula

$ax^2 + bx + c = 0$ has roots α and β -------(1)

It means $x = \alpha$ and $x = \beta$

This tells us that: $(x - \alpha)(x - \beta) = 0$

Expanding, we have: $x^2 - \alpha x - \beta x + \alpha\beta = 0$

$$\boxed{x^2 - (\alpha + \beta)x + \alpha\beta = 0}$$

So, if we compare the above with (1):

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Result: $\alpha\beta = \frac{c}{a}$, $\alpha + \beta = -\frac{b}{a}$

In other words…

$ax^2 + bx + c = 0$ has roots α and β

$$\boxed{x^2 - (\alpha + \beta)x + \alpha\beta = 0}$$

$$\boxed{x^2 - (\text{sum of roots})x + \text{product of roots} = 0}$$

Application – Example 1

$2x^2 + 6x - 3 = 0$ has roots α and β, find the value of

$$\frac{1}{\alpha} + \frac{1}{\beta}$$

Without finding α and β, we know the value for α + β and αβ

$\alpha + \beta = -3$ and $\alpha\beta = -3/2$

Hence,

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-3}{-\frac{3}{2}} = 2$$

$x^2 - 2x - 4 = 0$ has roots α and β, find the value of

$\alpha^2 + \beta^2$

Step 1: $\alpha + \beta = 2$ and $\alpha\beta = -4$

To find this, we use the algebraic property

$$(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$

Rearranging:

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= (2)^2 - 2(-4)$$

$$= 12$$

Independent Practice

$2x^2 + 4x - 1 = 0$ has roots α and β, form the equation with roots α^2 and β^2

Remember!

$$\boxed{x^2 - (\text{sum of roots})x + \text{product of roots} = 0}$$

$$\boxed{x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0}$$

So in order to get the equation, find $\alpha^2 + \beta^2$ and $\alpha^2\beta^2$

EXERCISE 2.2

Q1 :

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2-2x-8 \quad (\text{ii})\, 4s^2-4s+1 \quad (\text{iii})\, 6x^2-3-7x$

$(\text{iv})\, 4u^2+8u \quad (\text{v})\, t^2-15 \quad (\text{vi})\, 3x^2-x-4$

Answer :

$(\text{i}) \qquad x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e., when $x = 4$ or $x = - 2$

Therefore, the zeroes of x^2-2x-8 are 4 and - 2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$(\text{ii}) \qquad 4s^2-4s+1=(2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

$(\text{iii}) \qquad 6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$(\text{iv})\quad 4u^2+8u=4u^2+8u+0$

$=4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and - 2.

Sum of zeroes = $0+(-2)=-2=\frac{-(8)}{4}=\frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

$(\text{v})\quad t^2-15$

$=t^2-0t-15$

$=(t-\sqrt{15})(t+\sqrt{15})$

The value of $t^2 - 15$ is zero when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when

Q2 :

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

$(\text{i})\quad \frac{1}{4},-1\quad(\text{ii})\quad \sqrt{2},\frac{1}{3}\quad(\text{iii})\quad 0,\sqrt{5}$

$(\text{iv})\quad 1,1\quad(\text{v})\quad -\frac{1}{4},\frac{1}{4}\quad(\text{vi})\quad 4,1$

Answer :

$(\text{i})\quad \frac{1}{4},-1$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{1}{4}=\frac{-b}{a}$$

$$\alpha\beta=-1=\frac{-4}{4}=\frac{c}{a}$$

If $a=4$, then $b=-1$, $c=-4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2},\frac{1}{3}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\sqrt{2}=\frac{3\sqrt{2}}{3}=\frac{-b}{a}$$

$$\alpha\beta=\frac{1}{3}=\frac{c}{a}$$

If $a=3$, then $b=-3\sqrt{2}$, $c=1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0,\sqrt{5}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=0=\frac{0}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=\sqrt{5}=\frac{\sqrt{5}}{1}=\frac{c}{a}$$

If $a=1$, then $b=0$, $c=\sqrt{5}$

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$

$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$

If $a=1$, then $b=-1$, $c=1$

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4},\frac{1}{4}$

DIVISION ALGORITHM

Ways to divide a polynomial

Factor Theorem

- Use zero of a polynomial to find the other factor
- Factorize it to get the zeroes

Long Division

- Actual division of two polynomials
- Factorize the quotient to get the remaining zeroes

DIVISION ALGORITHM

Dividend = (Quotient × Divisor) + Remainder

Example 1:

Divide $2x^2 + 3x + 1$ by $x + 2$.

$$\begin{array}{r l} & \quad\;\; 2x - 1 \\ \cline{2-2} x + 2 \,\big) & 2x^2 + 3x + 1 \\ & \underline{\underset{-}{}2x^2 \underset{-}{+} 4x} \\ & \qquad -x + 1 \\ & \qquad \underline{\underset{+}{-}x \underset{+}{-} 2} \\ & \qquad\qquad\;\; 3 \\ \cline{2-2} \end{array}$$

Example 2:

Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

$$
\begin{array}{r|l}
 & x - 2 \\ \cline{2-2}
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & -x^3 + x^2 - x \\
 & + \quad\ \ - \quad\ \ + \\ \cline{2-2}
 & 2x^2 - 2x + 5 \\
 & 2x^2 - 2x + 2 \\
 & \quad\ \ + \quad\ \ - \\ \cline{2-2}
 & \qquad\quad 3
\end{array}
$$

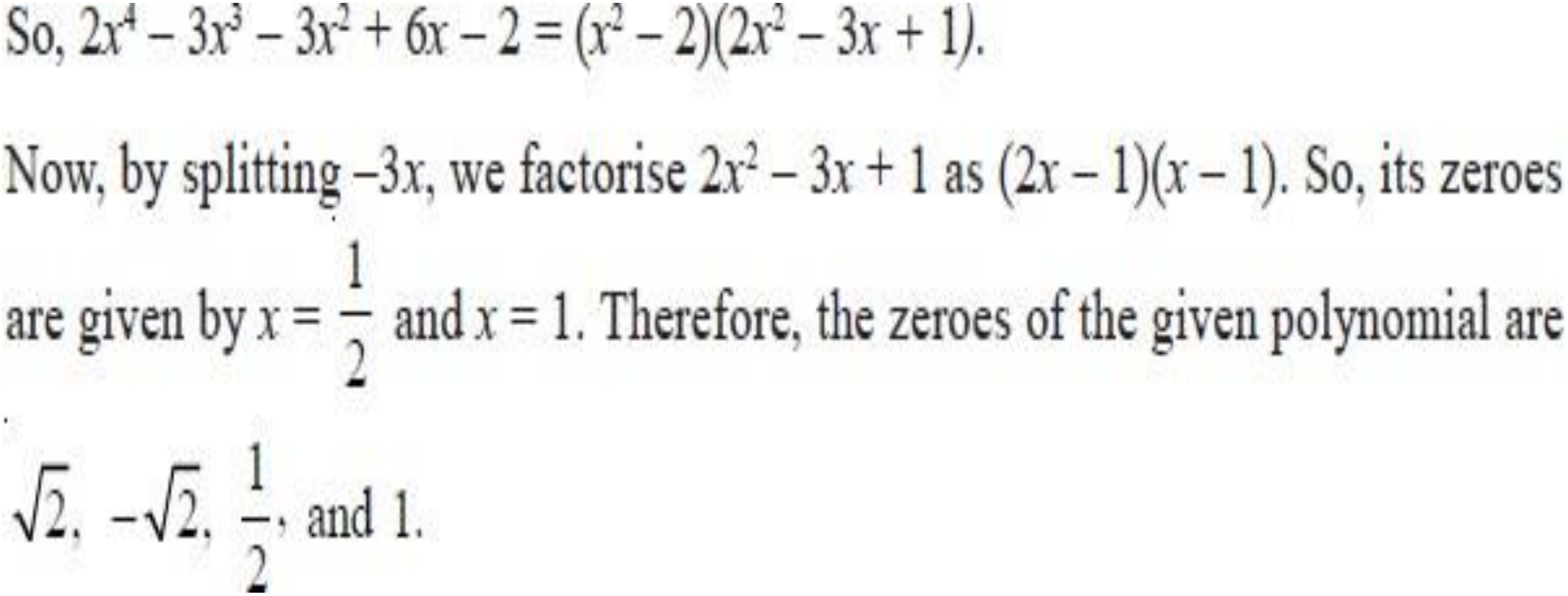
$$
\begin{aligned}
\text{Divisor} \times \text{Quotient} + \text{Remainder} \\
&= (-x^2 + x - 1)\,(x - 2) + 3 \\
&= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3 \\
&= -x^3 + 3x^2 - 3x + 5 \\
&= \text{Dividend}
\end{aligned}
$$

Example: 3

Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

$$
\begin{array}{r l}
 & 2x^2 - 3x + 1 \\ \cline{2-2}
x^2 - 2 \,\big) & 2x^4 - 3x^3 - 3x^2 + 6x - 2 \\
 & \underset{-}{2x^4} \qquad\quad \underset{+}{-4x^2} \\ \cline{2-2}
 & \qquad -3x^3 + x^2 + 6x - 2 \\
 & \qquad \underset{+}{-3x^3} \qquad \underset{-}{+6x} \\ \cline{2-2}
 & \qquad\qquad\quad x^2 \qquad -2 \\
 & \qquad\qquad\quad \underset{-}{x^2} \qquad \underset{+}{-2} \\ \cline{2-2}
 & \qquad\qquad\qquad\qquad\quad 0 \\ \cline{2-2}
\end{array}
$$

Example 3…contd..



So, $2x^4 - 3x^3 - 3x^2 + 6x - 2 = (x^2 - 2)(2x^2 - 3x + 1)$.

Now, by splitting $-3x$, we factorise $2x^2 - 3x + 1$ as $(2x - 1)(x - 1)$. So, its zeroes are given by $x = \frac{1}{2}$ and $x = 1$. Therefore, the zeroes of the given polynomial are

$\sqrt{2}$, $-\sqrt{2}$, $\frac{1}{2}$, and 1.

Exercise 2.3

Q1 :

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3, \qquad g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, \qquad g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6, \qquad g(x) = 2 - x^2$

Answer :

$$\begin{aligned} &(\text{i}) \qquad p(x) = x^3 - 3x^2 + 5x - 3 \\ &\qquad\quad\;\; q(x) = x^2 - 2 \end{aligned}$$

$$\begin{array}{r|llll}
 & x-3 & & & \\ \hline
x^2-2 & x^3 & -3x^2 & +5x & -3 \\
 & x^3 & & -2x & \\
 & - & & + & \\ \hline
 & & -3x^2 & +7x & -3 \\
 & & -3x^2 & & +6 \\
 & & + & & - \\ \hline
 & & & 7x & -9 \\ \hline
\end{array}$$

Quotient = x - 3

Remainder = 7x - 9

$$\begin{aligned} &(\text{ii}) \qquad p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5 \\ &\qquad\quad\;\;\; q(x) = x^2 + 1 - x = x^2 - x + 1 \end{aligned}$$

$$
\begin{array}{r l}
 & x^2 + x - 3 \\
x^2 - x + 1 \overline{)} & x^4 + 0.x^3 - 3x^2 + 4x + 5 \\
 & x^4 - \ \ x^3 + x^2 \\
 & - \quad + \qquad - \\
\hline
 & \qquad x^3 \ - 4x^2 + 4x + 5 \\
 & \qquad x^3 \ - \ x^2 + \ x \\
 & \qquad - \qquad + \quad - \\
\hline
 & \qquad\qquad -3x^2 + 3x + 5 \\
 & \qquad\qquad -3x^2 + 3x - 3 \\
 & \qquad\qquad + \qquad - \quad + \\
\hline
 & \qquad\qquad\qquad\qquad 8 \\
\hline
\end{array}
$$

Quotient = $x^2 + x - 3$

Remainder = 8

$$
\begin{array}{ll}
\text{(iii)} & p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6 \\
 & q(x) = 2 - x^2 = -x^2 + 2
\end{array}
$$

$$
\begin{array}{r l}
 & -x^2 - 2 \\
-x^2 + 2 \overline{)} & x^4 + 0.x^2 - 5x + 6 \\
 & x^4 - 2x^2 \\
 & - \quad + \\
\hline
 & \quad 2x^2 - 5x + 6 \\
 & \quad 2x^2 \qquad - 4 \\
 & \quad - \qquad\quad + \\
\hline
 & \qquad -5x + 10 \\
\hline
\end{array}
$$

Quotient = $- x^2 - 2$

Remainder = $- 5x + 10$

Q2 :

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

$$
\begin{array}{lll}
\text{(i)} & 2x^3 + x^2 - 5x + 2; & \frac{1}{2}, 1, -2 \\
\text{(ii)} & x^3 - 4x^2 + 5x - 2; & 2, 1, 1
\end{array}
$$

Answer :

(i) $p(x)=2x^3+x^2-5x+2.$

Zeroes for this polynomial are $\frac{1}{2},1,-2$

$$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$$

$$=\frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$$

$$=0$$

$$p(1)=2\times1^3+1^2-5\times1+2$$

$$=0$$

$$p(-2)=2(-2)^3+(-2)^2-5(-2)+2$$

$$=-16+4+10+2=0$$

Therefore, $\frac{1}{2}$, 1, and - 2 are the zeroes of the given polynomial.

Comparing the given polynomial with ax^3+bx^2+cx+d, we obtain $a = 2$, $b = 1$, $c = - 5$, $d = 2$

We can take $\alpha=\frac{1}{2},\beta=1,\gamma=-2$

$$\alpha+\beta+\gamma=\frac{1}{2}+1+(-2)=-\frac{1}{2}=\frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{1}{2}\times1+1(-2)+\frac{1}{2}(-2)=\frac{-5}{2}=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{1}{2}\times1\times(-2)=\frac{-1}{1}=\frac{-(2)}{2}=\frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x)=x^3-4x^2+5x-2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2)=2^3-4(2^2)+5(2)-2$$

$$=8-16+10-2=0$$

$$p(1)=1^3-4(1)^2+5(1)-2$$

$$=1-4+5-2=0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with ax^3+bx^2+cx+d, we obtain a = 1, b = - 4, c = 5, d = - 2.

Verification of the relationship between zeroes and coefficient of the given polynomial

$$\text{Sum of zeroes} = 2+1+1=4=\frac{-(-4)}{1}=\frac{-b}{a}$$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1) =2 + 1 + 2 = 5 $=\frac{(5)}{1}=\frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $=\frac{-(-2)}{1}=\frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Q3 :

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2-3, 2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

(iii) $x^3-3x+1, x^5-4x^3+x^2+3x+1$

Answer :

(i) $t^2-3,\ 2t^4+3t^3-2t^2-9t-12$

t^2-3 = $t^2+0.t-3$

$$\begin{array}{r|l}
 & 2t^2+3t+4 \\ \hline
t^2+0.t-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4+0.t^3-6t^2 \\
 & - \quad - \quad + \\ \cline{2-2}
 & \qquad 3t^3+4t^2-9t-12 \\
 & \qquad 3t^3+0.t^2-9t \\
 & \qquad - \quad - \quad + \\ \cline{2-2}
 & \qquad\qquad 4t^2+0.t-12 \\
 & \qquad\qquad 4t^2+0.t-12 \\
 & \qquad\qquad - \quad - \quad + \\ \cline{2-2}
 & \qquad\qquad\qquad 0 \\ \cline{2-2}
\end{array}$$

Since the remainder is 0,

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$\begin{array}{r|l}
 & \quad 3x^2-4x+2 \\ \hline
x^2+3x+1 & \quad 3x^4+5x^3-7x^2+2x+2 \\
 & \quad 3x^4+9x^3+3x^2 \\
 & - \quad - \quad - \\ \hline
 & \qquad -4x^3-10x^2+2x+2 \\
 & \qquad -4x^3-12x^2-4x \\
 & \qquad + \quad + \quad + \\ \hline
 & \qquad\qquad 2x^2+6x+2 \\
 & \qquad\qquad 2x^2+6x+2 \\ \hline
 & \qquad\qquad\qquad 0 \\ \hline
\end{array}$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$\begin{array}{r|l}
 & \qquad x^2-1 \\ \hline
x^3-3x+1 & \quad x^5-4x^3+x^2+3x+1 \\
 & \quad x^5-3x^3+x^2 \\
 & - \quad + \quad - \\ \hline
 & \qquad -x^3 \qquad +3x+1 \\
 & \qquad -x^3 \qquad +3x-1 \\
 & \qquad + \qquad\quad - \quad + \\ \hline
 & \qquad\qquad\qquad 2 \\ \hline
\end{array}$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Q4 :

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, - 7, - 14 respectively.

Answer :

Let the polynomial be ax^3+bx^2+cx+d and the zeroes be $\alpha, \beta, \text{ and } \gamma$.

It is given that

$$\alpha+\beta+\gamma=\frac{2}{1}=\frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{-7}{1}=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{-14}{1}=\frac{-d}{a}$$

If *a* = 1, then *b* = - 2, *c* = - 7, *d* = 14

Hence, the polynomial is $x^3-2x^2-7x+14$.

Q5 :

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}} \text{ and } -\sqrt{\frac{5}{3}}$.

Answer :

$$p(x)=3x^4+6x^3-2x^2-10x-5$$

Since the two zeroes are $\sqrt{\frac{5}{3}} \text{ and } -\sqrt{\frac{5}{3}}$,

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$
\begin{array}{r|l}
 & 3x^2+6x+3 \\
\hline
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & - \quad - \quad + \\
\hline
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3+0x^2-10x \\
 & \qquad - \quad - \quad + \\
\hline
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad - \quad - \quad + \\
\hline
 & \qquad\qquad\qquad 0 \\
\hline
\end{array}
$$

$$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$$

$$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$$

We factorize x^2+2x+1

$=\left(x+1\right)^2$

Therefore, its zero is given by $x + 1 = 0$

$x = - 1$

As it has the term $\left(x+1\right)^2$, therefore, there will be 2 zeroes at $x = - 1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, - 1 and - 1.

Q6 :

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were x - 2 and - $2x$ + 4, respectively. Find $g(x)$.

Answer :

$p\left(x\right)=x^3-3x^2+x+2 \qquad \left(\text{Dividend}\right)$

$g(x)$ = ? (Divisor)

Quotient = (x - 2)

Remainder = (- 2*x* + 4)

Dividend = Divisor × Quotient + Remainder

$$x^3-3x^2+x+2=g\left(x\right)\times\left(x-2\right)+\left(-2x+4\right)$$

$$x^3-3x^2+x+2+2x-4=g\left(x\right)\left(x-2\right)$$

$$x^3-3x^2+3x-2=g\left(x\right)\left(x-2\right)$$

g(*x*) is the quotient when we divide $\left(x^3-3x^2+3x-2\right)$ by $\left(x-2\right)$

$$\begin{array}{r l}
 & x^2-x+1 \\ \cline{2-2}
x-2\,\big) & x^3-3x^2+3x-2 \\
 & x^3-2x^2 \\
 & -\quad + \\ \cline{2-2}
 & \quad -x^2+3x-2 \\
 & \quad -x^2+2x \\
 & \quad +\quad - \\ \cline{2-2}
 & \qquad\qquad x-2 \\
 & \qquad\qquad x-2 \\
 & \qquad\qquad -\quad + \\ \cline{2-2}
 & \qquad\qquad 0 \\ \cline{2-2}
\end{array}$$

$$\therefore g\left(x\right)=\left(x^2-x+1\right)$$

Q7 :

Give examples of polynomial *p*(*x*), *g*(*x*), *q*(*x*) and *r*(*x*), which satisfy the division algorithm and

(i) deg *p*(*x*) = deg *q*(*x*)

(ii) deg *q*(*x*) = deg *r*(*x*)

(iii) deg *r*(*x*) = 0

Answer :

According to the division algorithm, if *p*(*x*) and *g*(*x*) are two polynomials with

g(*x*) ≠ 0, then we can find polynomials *q*(*x*) and *r*(*x*) such that

p(*x*) = *g*(*x*) × *q*(*x*) + *r*(*x*),

where *r*(*x*) = 0 or degree of *r*(*x*) < degree of *g*(*x*)

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2+2x+2$ by 2.

Here, $p(x) = 6x^2+2x+2$

$g(x) = 2$

$q(x) = 3x^2+x+1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2+2x+2 = 2(3x^2+x+1)$

$= 6x^2+2x+2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Q1 :

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a-b, a, a+b$, find *a* and *b*.

Answer :

$p(x) = x^3 - 3x^2 + x + 1$

Zeroes are $a - b, a + a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$p = 1, q = -3, r = 1, t = 1$

$\text{Sum of zeroes} = a - b + a + a + b$

$\frac{-q}{p} = 3a$

$\frac{-(-3)}{1} = 3a$

$3 = 3a$

$a = 1$

The zeroes are $1-b, 1, 1+b$.

$\text{Multiplication of zeroes} = 1(1-b)(1+b)$

$\frac{-t}{p} = 1 - b^2$

$\frac{-1}{1} = 1 - b^2$

$1 - b^2 = -1$

$1 + 1 = b^2$

$b = \pm\sqrt{2}$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

What have we learnt so far…..

Zeroes of a polynomial

- Geometrical representation

Relationship between zeroes and coefficients

- Sum and product of zeroes
- Forming a polynomial

Division Algorithm

- Long Division
- Finding all zeroes of the given polynomial

Just a closing thought....

"Do not worry about your difficulties in Mathematics. I assure you, mine are still greater..."

---- Albert Einstein

DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER 12 PROBABILITY

MIND MAP

The most probable questions from the examination point of view are given below.

Q.1. Red queens and black jacks are removed from a pack of 52 playing cards. A card is drawn at random from the remaining card, after reshuffling them. Find the probability that the drawn card is (i) a king (ii) of red colour (iii) a face card
(iv) queen.

Q.2. All the red face cards are removed from a pack of 52 playing cards. A card is drawn at random from the remaining cards after reshuffling them. Find the probability that the card drawn is (i) of red colour (ii) a queen (iii) an ace (iv) a face card.

Q.3. In a family of 3 children, find the probability of having at least 1 boy.

Q.4. Three unbiased coins are thrown simultaneously. Find the probability of getting
(i) Exactly two heads. (ii). At least two heads.
(iii) At most two heads.

Q.5. JKLM is a square with sides of length 6 units. Points A and B are the mid- points of sides KL and LM respectively. If a point is selected at random from the interior of the square, find the probability that the point will be chosen from the interior of ΔJAB.

Q.6. A square dart board is shown. The length of a side of the larger square is 1.5 times the length of a side of the smaller square. If a dart is thrown and lands on the larger square. What is the probability that it will land in the interior of the smaller square?

Q.7. From a pack of 52 playing cards Jacks, queens, kings and aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is
(i) a black queen. (ii) a red card. (iii) a black jack .
(iv) a picture card (Jacks, queens and kings are picture cards).

NCERT Solutions for Class 10 Maths Unit 15

Probability Class 10

Unit 15 Probability Exercise 15.1, 15.1 15.2, 15.2 Solutions

Exercise 15.1 : Solutions of Questions on Page Number : **308**

Q1 :

Complete the following statements:

(i) Probability of an event E + Probability of the event 'not E' = _______.

(ii) The probability of an event that cannot happen is _________. Such as event is called _________.

(iii) The probability of an event that is certain to happen is _________. Such as event is called ________.

(iv) The sum of the probabilities of all the elementary events of an experiment is _________.

(v) The probability of an event is greater than or equal to _______ and less than or equal to _______.

Answer :

(i) 1

(ii) 0, impossible event

(iii) 1, sure event or certain event

(iv) 1

(v) 0, 1

Q2 :

Which of the following experiments have equally likely outcomes? Explain.

(i) A driver attempts to start a car. The car starts or does not start.

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A trial is made to answer a true-false question. The answer is right or wrong.

(iv) A baby is born. It is a boy or a girl.

Answer :

(i) It is not an equally likely event, as it depends on various factors such as whether the car will start or not. And factors for both the conditions are not the same.

(ii) It is not an equally likely event, as it depends on the player's ability and there is no information given about that.

(iii) It is an equally likely event.

(iv) It is an equally likely event.

Q3 :

Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Answer :

When we toss a coin, the possible outcomes are only two, head or tail, which are equally likely outcomes. Therefore, the result of an individual toss is completely unpredictable.

Q4 :

Which of the following cannot be the probability of an event?

(A) $\frac{2}{3}$ (B) -1.5 (C) 15% (D) 0.7

Answer :

Probability of an event (E) is always greater than or equal to 0. Also, it is always less than or equal to one. This implies that the probability of an event cannot be negative or greater than 1. Therefore, out of these alternatives, -1.5 cannot be a probability of an event.

Hence, (B)

Q5 :

If P(E) = 0.05, what is the probability of 'not E'?

Answer :

We know that,

$$
\begin{aligned}
P(\overline{E}) &= 1 - P(E) \\
P(\overline{E}) &= 1 - 0.05 \\
&= 0.95
\end{aligned}
$$

Therefore, the probability of 'not E' is 0.95.

Q6 :

A bag contains lemon flavoured candies only. Malini takes out one candy without looking into the bag. What is the probability that she takes out

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Answer :

(i) The bag contains lemon flavoured candies only. It does not contain any orange flavoured candies. This implies that every time, she will take out only lemon flavoured candies. Therefore, event that Malini will take out an orange flavoured candy is an impossible event.

Hence, P (an orange flavoured candy) = 0

(ii)As the bag has lemon flavoured candies, Malini will take out only lemon flavoured candies. Therefore, event that Malini will take out a lemon flavoured candy is a sure event.

P (a lemon flavoured candy) = 1

Q7 :

It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Answer :

Probability that two students are not having same birthday P ($\overline{E}$) = 0.992

Probability that two students are having same birthday P (E) = 1 - P ($\overline{E}$)

= 1 - 0.992

= 0.008

Q8 :

A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is (i) red? (ii) not red?

Answer :

(i) Total number of balls in the bag = 8

$$\text{Probability of getting a red ball} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{3}{8}$$

(ii) Probability of not getting red ball

= 1 - Probability of getting a red ball

$$= 1 - \frac{3}{8}$$

$$= \frac{5}{8}$$

Q9 :

A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white? (iii) not green?

Answer :

Total number of marbles = 5 + 8 + 4

= 17

(i)Number of red marbles = 5

$$\text{Probability of getting a red marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{5}{17}$$

(ii)Number of white marbles = 8

$$\text{Probability of getting a white marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{8}{17}$$

(iii)Number of green marbles = 4

$$\text{Probability of getting a green marble} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{4}{17}$$

Probability of not getting a green marble $= 1 - \frac{4}{17} = \frac{13}{17}$

Q10 :

A piggy bank contains hundred 50 p coins, fifty Rs 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin

(i) Will be a 50 p coin?

(ii) Will not be a Rs.5 coin?

Answer :

Total number of coins in a piggy bank = 100 + 50 + 20 + 10

= 180

(i) Number of 50 p coins = 100

$$\text{Probability of getting a 50 p coin} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{100}{180} = \frac{5}{9}$$

(ii) Number of Rs 5 coins = 10

$$\text{Probability of getting a Rs 5 coin} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{10}{180} = \frac{1}{18}$$

Probability of not getting a Rs 5 coin $= 1 - \frac{1}{18}$

$$= \frac{17}{18}$$

Q11 :

Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fish and 8 female fish (see the given figure). What is the probability that the fish taken out is a male fish?

Answer :

Total number of fishes in a tank

= Number of male fishes + Number of female fishes

= 5 + 8 = 13

$$\text{Probability of getting a male fish} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{5}{13}$$

Q12 :

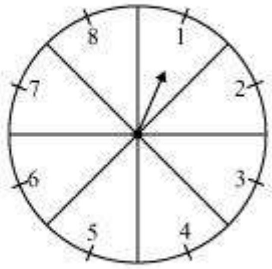
A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see the given figure), and these are equally likely outcomes. What is the probability that it will point at

(i) 8?

(ii) an odd number?

(iii) a number greater than 2?

(iv) a number less than 9?



Answer :

Total number of possible outcomes = 8

(i) $$\text{Probability of getting 8} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \frac{1}{8}$$

(ii) Total number of odd numbers on spinner = 4

$$\text{Probability of getting an odd number} = \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{4}{8} = \frac{1}{2}$$

(iii) The numbers greater than 2 are 3, 4, 5, 6, 7, and 8.

Therefore, total numbers greater than 2 = 6

$$\text{Probability of getting a number greater than 2}$$

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}} = \frac{6}{8} = \frac{3}{4}$$

(iv) The numbers less than 9 are 1, 2, 3, 4, 6, 7, and 8.

Therefore, total numbers less than 9 = 8

Probability of getting a number less than 9 $= \frac{8}{8} = 1$

Q13 :

A die is thrown once. Find the probability of getting

(i) a prime number;

(ii) a number lying between 2 and 6;

(iii) an odd number.

Answer :

The possible outcomes when a dice is thrown = {1, 2, 3, 4, 5, 6}

Number of possible outcomes of a dice = 6

(i) Prime numbers on a dice are 2, 3, and 5.

Total prime numbers on a dice = 3

Probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$

(ii) Numbers lying between 2 and 6 = 3, 4, 5

Total numbers lying between 2 and 6 = 3

Probability of getting a number lying between 2 and 6 $= \frac{3}{6} = \frac{1}{2}$

(iii) Odd numbers on a dice = 1, 3, and 5

Total odd numbers on a dice = 3

Probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$

Q14 :

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour

(ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds

Answer :

Total number of cards in a well-shuffled deck = 52

(i) Total number of kings of red colour = 2

P (getting a king of red colour) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{2}{52} = \frac{1}{26}$

(ii) Total number of face cards = 12

P (getting a face card) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{12}{52} = \frac{3}{13}$

(iii) Total number of red face cards = 6

P (getting a red face card) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{6}{52} = \frac{3}{26}$

(iv) Total number of Jack of hearts = 1

P (getting a Jack of hearts) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{1}{52}$

(v) Total number of spade cards = 13

P (getting a spade card) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{13}{52} = \frac{1}{4}$

(vi) Total number of queen of diamonds = 1

P (getting a queen of diamond) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{1}{52}$

Q15 :

Five cards--the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Answer :

(i) Total number of cards = 5

Total number of queens = 1

P (getting a queen) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$= \frac{1}{5}$

(ii) When the queen is drawn and put aside, the total number of remaining cards will be 4.

(a) Total number of aces = 1

P (getting an ace) $= \frac{1}{4}$

(b) As queen is already drawn, therefore, the number of queens

will be 0.

P (getting a queen) $= \frac{0}{4}$ = 0

Q16 :

12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Answer :

Total number of pens = 12 + 132 = 144

Total number of good pens = 132

P (getting a good pen) $= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$$=\frac{132}{144}=\frac{11}{12}$$

Q17 :

(i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Answer :

(i) Total number of bulbs = 20

Total number of defective bulbs = 4

P (getting a defective bulb) $=\frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$

$$=\frac{4}{20}=\frac{1}{5}$$

(ii) Remaining total number of bulbs = 19

Remaining total number of non-defective bulbs = 16 - 1 = 15

P (getting a not defective bulb) $=\frac{15}{19}$

Q18 :

A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two-digit number

(ii) a perfect square number

(iii) a number divisible by 5.

Answer :

Total number of discs = 90

(i) Total number of two-digit numbers between 1 and 90 = 81

P (getting a two-digit number) $=\frac{81}{90}=\frac{9}{10}$

(ii) Perfect squares between 1 and 90 are 1, 4, 9, 16, 25, 36, 49, 64, and 81. Therefore, total number of perfect squares between 1 and 90 is 9.

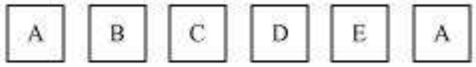
P (getting a perfect square) $= \frac{9}{90} = \frac{1}{10}$

(iii) Numbers that are between 1 and 90 and divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, and 90. Therefore, total numbers divisible by 5 = 18

Probability of getting a number divisible by 5 $= \frac{18}{90} = \frac{1}{5}$

Q19 :

A child has a die whose six faces shows the letters as given below:



The die is thrown once. What is the probability of getting (i) A? (ii) D?

Answer :

Total number of possible outcomes on the dice = 6

(i) Total number of faces having A on it = 2

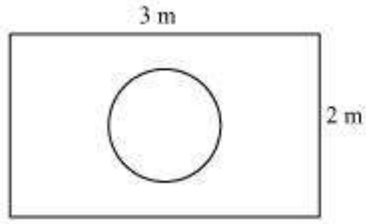
P (getting A) $= \frac{2}{6} = \frac{1}{3}$

(ii) Total number of faces having D on it = 1

P (getting D) $= \frac{1}{6}$

Q20 :

Suppose you drop a die at random on the rectangular region shown in the given figure. What is the probability that it will land inside the circle with diameter 1 m?



Answer :

Area of rectangle = *l* × *b* = 3 × 2 = 6 m^2

Area of circle (of diameter 1 m) $= \pi r^2 = \pi\left(\frac{1\pi}{2}\right)^2 = \frac{}{4}$ m^2

P (die will land inside the circle) $= \frac{\frac{\pi}{4}}{6} = \frac{\pi}{24}$

Q21 :

A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that

(i) She will buy it?

(ii) She will not buy it?

Answer :

Total number of pens = 144

Total number of defective pens = 20

Total number of good pens = 144 - 20 = 124

(i) Probability of getting a good pen $= \frac{124}{144} = \frac{31}{36}$

P (Nuri buys a pen) $= \frac{31}{36}$

(ii) P (Nuri will not buy a pen) $= 1 - \frac{31}{36} = \frac{5}{36}$

Q22 :

Two dice, one blue and one grey, are thrown at the same time.

(i) Write down all the possible outcomes and complete the following table:

Event: **Sum of two dice**	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore, each of them has a probability $\frac{1}{11}$

Answer :

(i) It can be observed that,

To get the sum as 2, possible outcomes = (1, 1)

To get the sum as 3, possible outcomes = (2, 1) and (1, 2)

To get the sum as 4, possible outcomes = (3, 1), (1, 3), (2, 2)

To get the sum as 5, possible outcomes = (4, 1), (1, 4), (2, 3), (3, 2)

To get the sum as 6, possible outcomes = (5, 1), (1, 5), (2, 4), (4, 2), (3, 3)

To get the sum as 7, possible outcomes = (6, 1), (1, 6), (2, 5), (5, 2), (3, 4), (4, 3)

To get the sum as 8, possible outcomes = (6, 2), (2, 6), (3, 5), (5, 3), (4, 4)

To get the sum as 9, possible outcomes = (3, 6), (6, 3), (4, 5), (5, 4)

To get the sum as 10, possible outcomes = (4, 6), (6, 4), (5, 5)

To get the sum as 11, possible outcomes = (5, 6), (6, 5)

To get the sum as 12, possible outcomes = (6, 6)

Event: **Sum of two dice**	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii)Probability of each of these sums will not be $\frac{1}{11}$ as these sums are not equally likely.

Q23 :

A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result i.e., three heads or three tails, and loses otherwise. Calculate the probability that Hanif will lose the game.

Answer :

The possible outcomes are

{HHH, TTT, HHT, HTH, THH, TTH, THT, HTT}

Number of total possible outcomes = 8

Number of favourable outcomes = 2 {i.e., TTT and HHH}

P (Hanif will win the game) $= \frac{2}{8} = \frac{1}{4}$

P (Hanif will lose the game) $= 1 - \frac{1}{4} \; = \frac{3}{4}$

Q24 :

A die is thrown twice. What is the probability that

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

[Hint: Throwinga die twice and throwing two dice simultaneously are treated as the same experiment].

Answer :

Total number of outcomes = 6 × 6

= 36

(i)Total number of outcomes when 5 comes up on either time are (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (1, 5), (2, 5), (3, 5), (4, 5), (6, 5)

Hence, total number of favourable cases = 11

P (5 will come up either time) $= \frac{11}{36}$

P (5 will not come up either time) $= 1 - \frac{11}{36} \; = \frac{25}{36}$

(ii)Total number of cases, when 5 can come at least once = 11

P (5 will come at least once) $= \frac{11}{36}$

Exercise 15.1 15.2 : Solutions of Questions on Page Number : **311**

Q1 :

Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on

(i) the same day? (ii) consecutive days? (iii) different days?

Answer :

There are a total of 5 days. Shyam can go to the shop in 5 ways and Ekta can go to the shop in 5 ways.

Therefore, total number of outcomes = 5 × 5 = 25

(i) They can reach on the same day in 5 ways.

i.e., (t, t), (w, w), (th, th), (f, f), (s, s)

P (both will reach on same day) $= \frac{5}{25}$ $= \frac{1}{5}$

(ii) They can reach on consecutive days in these 8 ways - (t, w), (w, th), (th, f), (f, s), (w, t), (th, w), (f, th), (s, f).

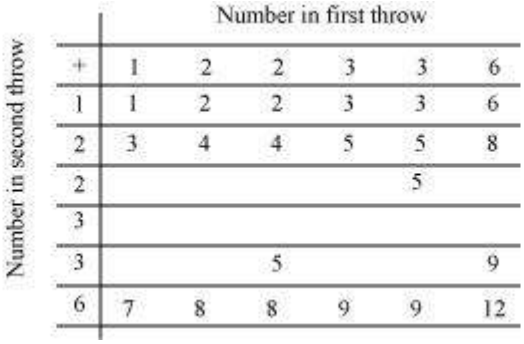
Therefore, P (both will reach on consecutive days) $= \frac{8}{25}$

(iii) P (both will reach on same day) $= \frac{1}{5}$ [(From (i)]

P (both will reach on different days) $= 1 - \frac{1}{5} = \frac{4}{5}$

Q2 :

A die is numbered in such a way that its faces show the number 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:



		Number in first throw					
	+	1	2	2	3	3	6
Number in second throw	1	1	2	2	3	3	6
	2	3	4	4	5	5	8
	2					5	
	3						
	3			5			9
	6	7	8	8	9	9	12

What is the probability that the total score is

(i) even? (ii) 6? (iii) at least 6?

Answer :

+	1 2 2 3 3 6
1	2 3 3 4 4 7

2	3 4 4 5 5 8
2	3 4 4 5 5 8
3	4 5 5 6 6 9
3	4 5 5 6 6 9
6	7 8 8 9 9 12

Total number of possible outcomes when two dice are thrown = 6 × 6 = 36

(i) Total times when the sum is even = 18

P (getting an even number) $= \frac{18}{36} = \frac{1}{2}$

(ii) Total times when the sum is 6 = 4

P (getting sum as 6) $= \frac{4}{36} = \frac{1}{9}$

(iii) Total times when the sum is at least 6 (i.e., greater than 5) = 15

P (getting sum at least 6) $= \frac{15}{36} = \frac{5}{12}$

Q3 :

A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is *double* that of a red ball, determine the number of blue balls in the bag.

Answer :

Let the number of blue balls be x.

Number of red balls = 5

Total number of balls = $x + 5$

P (getting a red ball) $= \frac{5}{x+5}$

P (getting a blue ball) $= \frac{x}{x+5}$

Given that,

$$2\left(\frac{5}{x+5}\right)=\frac{x}{x+5}$$
$$10(x+5)=x^2+5x$$
$$x^2-5x-50=0$$
$$x^2-10x+5x-50=0$$
$$x(x-10)+5(x-10)=0$$
$$(x-10)(x+5)=0$$
$$\text{Either } x-10=0 \text{ or } x+5=0$$
$$x=10 \text{ or } x=-5$$

However, the number of balls cannot be negative.

Hence, number of blue balls = 10

Q4 :

A box contains 12 balls out of which *x* are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find *x*.

Answer :

Total number of balls = 12

Total number of black balls = x

P (getting a black ball) = $\frac{x}{12}$

If 6 more black balls are put in the box, then

Total number of balls = 12 + 6 = 18

Total number of black balls = x + 6

P (getting a black ball now) $=\frac{x+6}{18}$

According to the condition given in the question,

$$2\left(\frac{x}{12}\right)=\frac{x+6}{18}$$
$$3x=x+6$$
$$2x=6$$
$$x=3$$

Q5 :

Which of the following arguments are correct and which are not correct? Give reasons for your answer.

(i) If two coins are tossed simultaneously there are three possible outcomes - - two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii) If a die is thrown, there are two possible outcomes - - an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Answer :

(i) Incorrect

When two coins are tossed, the possible outcomes are (H, H), (H, T), (T, H), and (T, T). It can be observed that there can be one of each in two possible ways - (H, T), (T, H).

Therefore, the probability of getting two heads is $\frac{1}{4}$, the probability of getting two tails is $\frac{1}{4}$, and the probability of getting one of each is $\frac{1}{2}$.

It can be observed that for each outcome, the probability is not $\frac{1}{3}$.

(ii) Correct

When a dice is thrown, the possible outcomes are 1, 2, 3, 4, 5, and 6. Out of these, 1, 3, 5 are odd and 2, 4, 6 are even numbers.

Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Exercise 15.2 : Solutions of Questions on Page Number : **312**

Q1 :

A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

Answer :

Total number of marbles = 24

Let the total number of green marbles be *x*.

Then, total number of blue marbles = 24 - *x*

P (getting a given marble) $= \frac{x}{24}$

According to the condition given in the question,

$$\frac{x}{24} = \frac{2}{3}$$
$$x = 16$$

Therefore, total number of green marbles in the jar = 16

Hence, total number of blue marbles = 24 - *x* = 24 - 16 = 8

DELHI PUBLIC SCHOOL, GANDHINAGAR

MIND MAP

CH.3 PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

This chapter consists of three different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 REPRESENTING THE SITUATION ALGEBRAICALLY AND GRAPHICALLY

1. 7 audio cassettes and 3 video cassettes cost ₹1110, while 5 audio cassettes and 4 video cassettes cost ₹1350. Find the cost of an audio cassette and a video cassette

2. 3 bags and 4 pens together cost ₹257 whereas 4 bags and 3 pens together cost ₹324.

   Find the total cost of 1 bag and 10 pens.

TYPE: 2 GRAPHICAL METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

1. Draw the graph of the equation$4x + 5y = 12$.

   Determine the vertices of the triangle formed by these lines and axes.

2. Solve the following system of linear equations graphically: $x + 2y - 7 = 0; 2x - y - 4 = 0$. Shade the area bounded by the lines and y-axis.

TYPE: 3 ALGEBRAIC METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

1. Solve the given equations by substitution, elimination and cross multiplication method.

   $ax + by = a - b\,; \quad bx - ay = a + b.$

2. $7(y + 3) - 2(x + 3) = 14; \;\; 4(y - 2) + 3(x - 3) = 2$

3. $2x - \frac{3}{y} = 9; \quad 3x + \frac{7}{y} = 2$

NCERT Solutions for Class 10 Maths Unit 3

Pair of Linear Equations in Two Variables Class 10

Unit 3 Pair of Linear Equations in Two Variables Exercise 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7 Solutions

If two linear equations have the two same variables, they are called a pair of linear equations in two variables. Following is the most general form of linear equations:

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Here, a1, a2, b1, b2, c1 and c2 are real numbers such that;

$$a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$$

A pair of linear equations can be represented and solved by the following methods:

a. Graphical method
b. Algebraic method

Exercise 3.1 : Solutions of Questions on Page Number : **44**

Q1 :

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Answer :

Let the present age of Aftab be *x*.

And, present age of his daughter = *y*

Seven years ago,

Age of Aftab = *x* - 7

Age of his daughter = *y* - 7

According to the question,

$$\begin{aligned}(x-7) &= 7(y-7)\\ x-7 &= 7y-49\\ x-7y &= -42 \qquad (1)\end{aligned}$$

Three years hence,

Age of Aftab = *x* + 3

Age of his daughter = *y* + 3

According to the question,

$$(x+3)=3(y+3)$$
$$x+3 \quad =3y+9$$
$$x-3y=6 \qquad\qquad (2)$$

Therefore, the algebraic representation is

$$x-7y=-42$$
$$x-3y=6$$

For $x-7y=-42$,

$$x=-42+7y$$

The solution table is

x	- 7	0	7
y	5	6	7

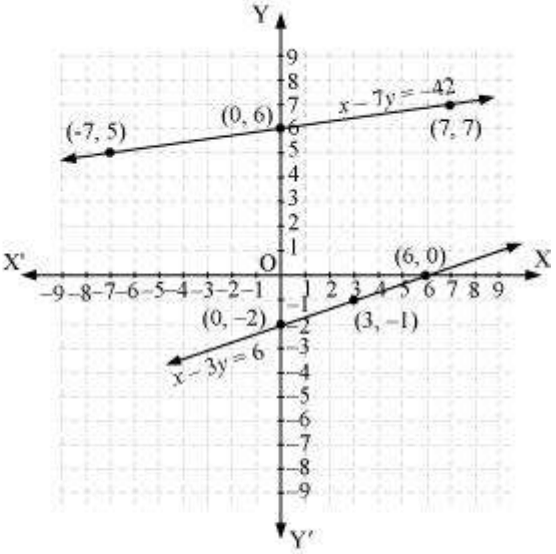
For $x-3y=6$,

$$x=6+3y$$

The solution table is

x	6	3	0
y	0	- 1	- 2

The graphical representation is as follows.



Q2 :

The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 3 more balls of the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Answer :

Let the cost of a bat be Rs *x*.

And, cost of a ball = Rs *y*

According to the question, the algebraic representation is

[Math Processing Error]

For $3x+6y=3900$,

$$x=\frac{3900-6y}{3}$$

The solution table is

x	300	100	-100
y	500	600	700

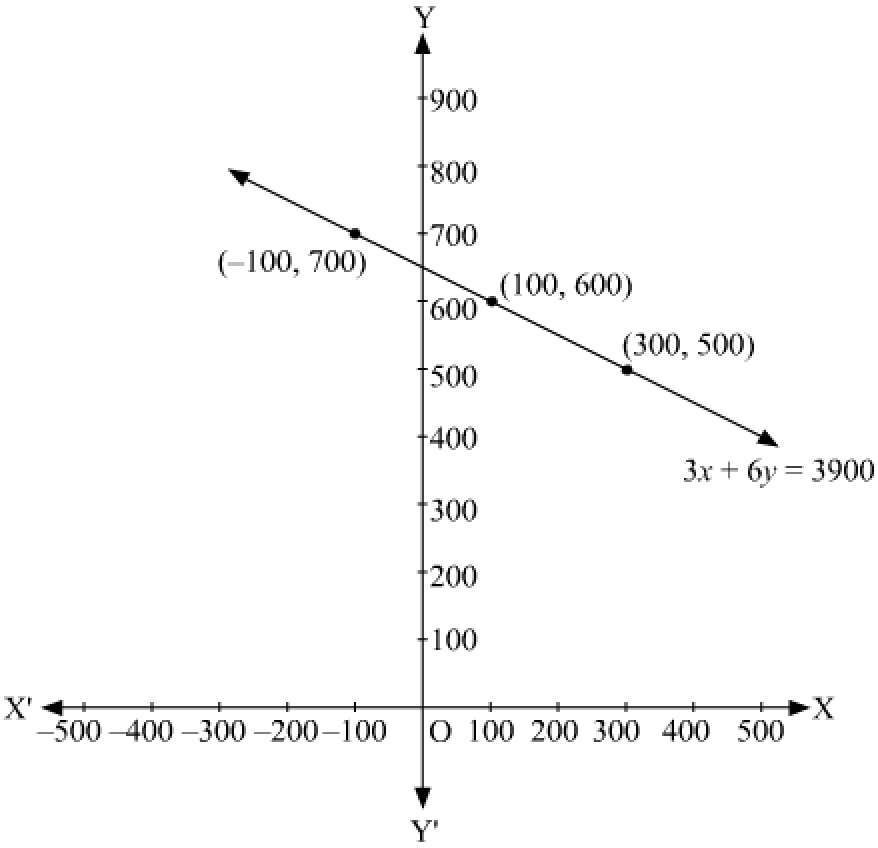
For x+ 3y = 1300,

x = 1300 - 2y

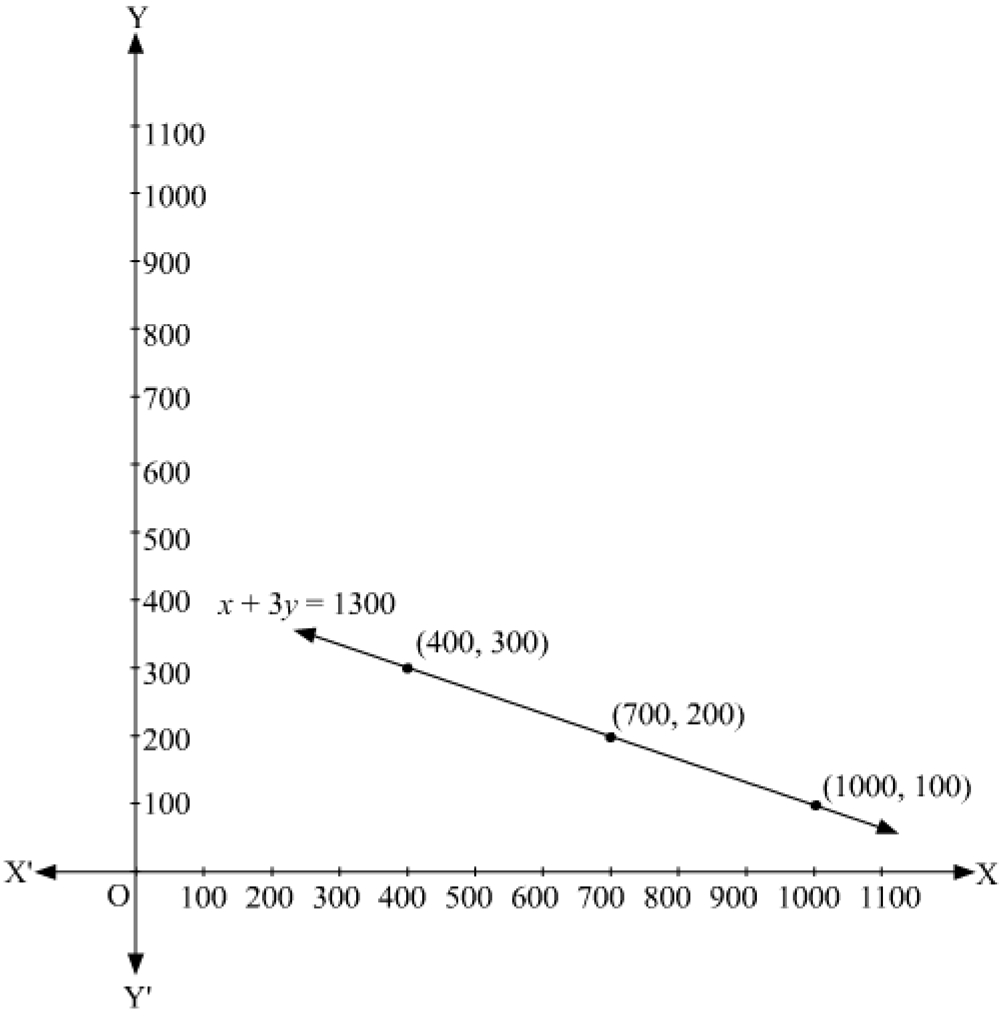
The solution table is

x	400	700	1000
y	300	200	100

The graphical representation for first line is as follows.



And graph for second line will be,



Q3 :

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer :

Let the cost of 1 kg of apples be Rs *x*.

And, cost of 1 kg of grapes = Rs *y*

According to the question, the algebraic representation is

$$2x + y = 160$$
$$4x + 2y = 300$$

For $2x + y = 160$,

$$y = 160 - 2x$$

The solution table is

x	50	60	70
y	60	40	20

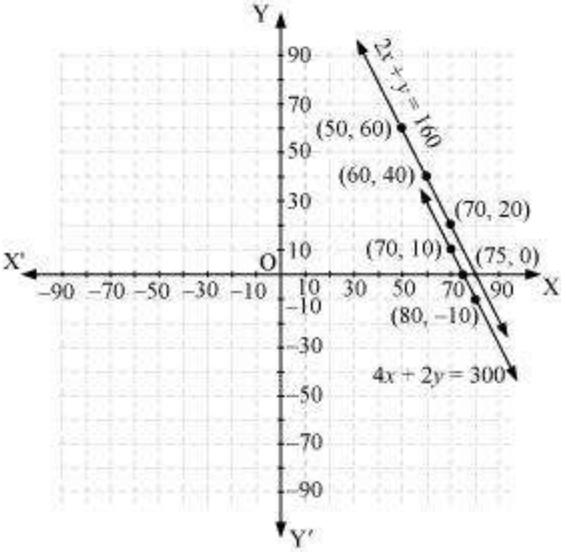
For $4x + 2y = 300$,

$$y = \frac{300 - 4x}{2}$$

The solution table is

x	70	80	75
y	10	- 10	0

The graphical representation is as follows.



Exercise 3.2 : Solutions of Questions on Page Number : **49**

Q1 :

Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Answer :

(i) Let the number of girls be x and the number of boys be y.

According to the question, the algebraic representation is

$x + y = 10$

$x - y = 4$

For $x + y = 10$,

$x = 10 - y$

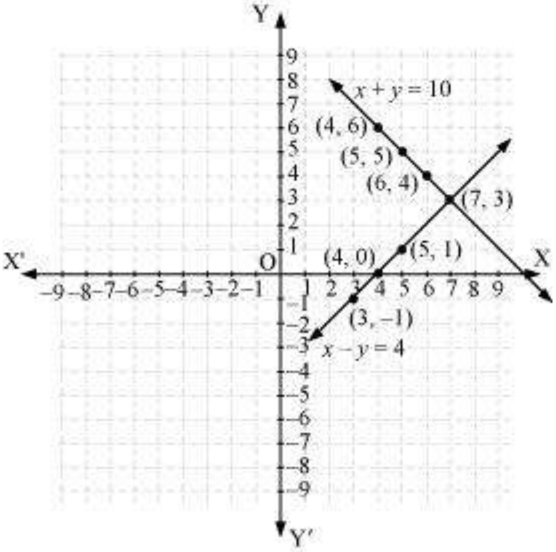
x	5	4	6
y	5	6	4

For $x - y = 4$,

$x = 4 + y$

x	5	4	3
y	1	0	- 1

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (7, 3).

Therefore, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of 1 pencil be Rs x and the cost of 1 pen be Rs y.

According to the question, the algebraic representation is

$5x + 7y = 50$

$7x + 5y = 46$

For $5x + 7y = 50$,

$$x = \frac{50 - 7y}{5}$$

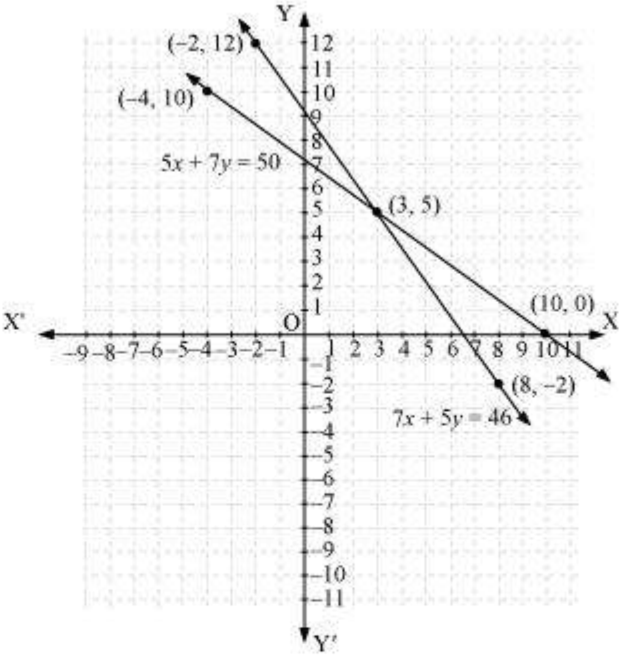
x	3	10	- 4
y	5	0	10

$7x + 5y = 46$

$$x = \frac{46 - 5y}{7}$$

x	8	3	- 2
y	- 2	5	12

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are Rs 3 and Rs 5 respectively.

Q2 :

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ **, find out whether the lines representing the following pairs of linear equations at a point, are parallel or coincident:**

$$\text{(i)}\ \begin{array}{l} 5x - 4y + 8 = 0 \\ 7x + 6y - 9 = 0 \end{array} \quad \text{(ii)}\ \begin{array}{l} 9x + 3y + 12 = 0 \\ 18x + 6y + 24 = 0 \end{array} \quad \text{(iii)}\ \begin{array}{l} 6x - 3y + 10 = 0 \\ 2x - y + 9 = 0 \end{array}$$

Answer :

(i) 5*x* - 4*y* + 8 = 0

7*x* + 6*y* - 9 = 0

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$$\begin{aligned} &a_1 = 5, \quad b_1 = -4, \quad c_1 = 8 \\ &a_2 = 7, \quad b_2 = 6, \quad c_2 = -9 \\ &\frac{a_1}{a_2} = \frac{5}{7} \\ &\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3} \end{aligned}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point.

(ii) 9*x* + 3*y* + 12 = 0

18*x* + 6*y* + 24 = 0

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$, we obtain

$$\begin{aligned} &a_1 = 9, \quad b_1 = 3, \quad c_1 = 12 \\ &a_2 = 18, \quad b_2 = 6, \quad c_2 = 24 \\ &\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2} \\ &\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2} \\ &\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2} \end{aligned}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Hence, the lines representing the given pair of equations are coincident and there are infinite possible solutions for the given pair of equations.

(iii)6*x* - 3*y* + 10 = 0

2*x* - *y* + 9 = 0

Comparing these equations with $a_1x + b_1y + c_1 = 0$

and $a_2x+b_2y+c_2=0$, we obtain

$$a_1=6, \quad b_1=-3, \quad c_1=10$$
$$a_2=2, \quad b_2=-1, \quad c_2=9$$

$$\frac{a_1}{a_2}=\frac{6}{2}=\frac{3}{1}$$
$$\frac{b_1}{b_2}=\frac{-3}{-1}=\frac{3}{1}$$
$$\frac{c_1}{c_2}=\frac{10}{9}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$,

Hence, the lines representing the given pair of equations are parallel to each other and hence, these lines will never intersect each other at any point or there is no possible solution for the given pair of equations.

Q3 :

On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

$$\text{(i)}\ 3x+2y=5; \quad 2x-3y=7 \qquad \text{(ii)}\ 2x-3y=8; \quad 4x-6y=9$$
$$\text{(iii)}\ \frac{3}{2}x+\frac{5}{3}y=7\ ; \quad 9x-10y=14 \qquad \text{(iv)}\ 5x-3y=11\ ;-10x+6y=-22$$
$$\text{(v)}\ \frac{4}{3}x+2y=8\ ; \quad 2x+3y=12$$

Answer :

(i) $3x + 2y = 5$

$2x - 3y = 7$

$$\frac{a_1}{a_2}=\frac{3}{2}, \quad \frac{b_1}{b_2}=\frac{-2}{3}, \quad \frac{c_1}{c_2}=\frac{5}{7}$$
$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

These linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(ii)$2x - 3y = 8$

4*x* - 6*y* = 9

$$\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2},\quad \frac{b_1}{b_2}=\frac{-3}{-6}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{8}{9}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii) $\frac{3}{2}x+\frac{5}{3}y=7$

$$9x-10y=14$$

$$\frac{a_1}{a_2}=\frac{\frac{3}{2}}{9}=\frac{1}{6},\quad \frac{b_1}{b_2}=\frac{\frac{5}{3}}{-10}=\frac{-1}{6},\quad \frac{c_1}{c_2}=\frac{7}{14}=\frac{1}{2}$$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(iv)5*x* - 3 *y* = 11

- 10*x* + 6*y* = - 22

$$\frac{a_1}{a_2}=\frac{5}{-10}=\frac{-1}{2},\quad \frac{b_1}{b_2}=\frac{-3}{6}=\frac{-1}{2},\quad \frac{c_1}{c_2}=\frac{11}{-22}=\frac{-1}{2}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

(v) $\frac{4}{3}x+2y=8$

$$2x+3y=12$$

$$\frac{a_1}{a_2}=\frac{\frac{4}{3}}{2}=\frac{2}{3},\quad \frac{b_1}{b_2}=\frac{2}{3},\quad \frac{c_1}{c_2}=\frac{8}{12}=\frac{2}{3}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

Q4 :

Which of the following pairs of linear equations are consistent/ inconsistent? If consistent, obtain the solution graphically:

(i) $x+y=5, \quad 2x+2y=10$

(ii) $x-y=8, \quad 3x-3y=16$

(iii) $2x+y-6=0, \quad 4x-2y-4=0$

(iv) $2x-2y-2=0, 4x-4y-5=0$

Answer :

(i)$x + y = 5$

$2x + 2y = 10$

$$\frac{a_1}{a_2}=\frac{1}{2}, \quad \frac{b_1}{b_2}=\frac{1}{2}, \quad \frac{c_1}{c_2}=\frac{5}{10}=\frac{1}{2}$$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

$x + y = 5$

$x = 5 - y$

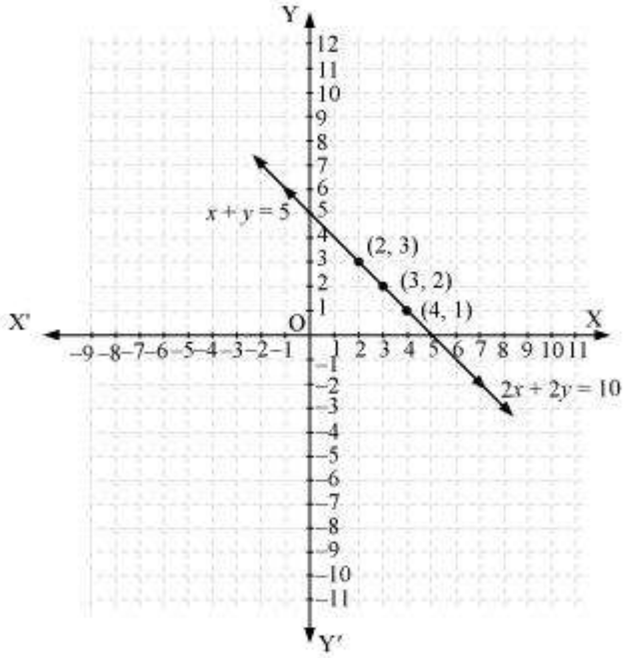
x	4	3	2
y	1	2	3

And, $2x + 2y = 10$

$$x=\frac{10-2y}{2}$$

x	4	3	2
y	1	2	3

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are overlapping each other. Therefore, infinite solutions are possible for the given pair of equations.

(ii)$x - y = 8$

$3x - 3y = 16$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii)$2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$2x + y - 6 = 0$

$y = 6 - 2x$

x	0	1	2
y	6	4	2

And $4x - 2y - 4 = 0$

$$y = \frac{4x - 4}{2}$$

x	1

Q5 :

Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Answer :

Let the width of the garden be x and length be y.

According to the question,

$y - x = 4$ (1)

$y + x = 36$ (2)

$y - x = 4$

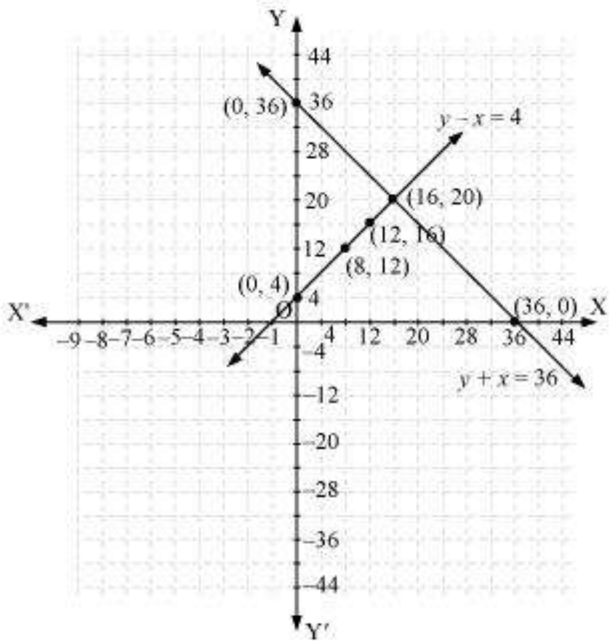
$y = x + 4$

x	0	8	12
y	4	12	16

$y + x = 36$

x	0	36	16
y	36	0	20

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at only point i.e., (16, 20). Therefore, the length and width of the given garden is 20 m and 16 m respectively.

Q6 :

Given the linear equation 2*x* + 3*y* - 8 = 0, write another linear equations in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines (ii) parallel lines

(iii) coincident lines

Answer :

(i)Intersecting lines:

For this condition,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The second line such that it is intersecting the given line is $2x + 4y - 6 = 0 \quad \text{as} \quad \frac{a_1}{a_2} = \frac{2}{2} = 1, \quad \frac{b_1}{b_2} = \frac{3}{4} \text{ and } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

(ii) Parallel lines:

For this condition,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the second line can be

$4x + 6y - 8 = 0$

$$\text{as}\ \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-8}{-8} = 1$$

$$\text{And clearly,} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(iii)<u>Coincident lines:</u>

For coincident lines,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the second line can be

$6x + 9y - 24 = 0$

$$\text{as}\ \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$$

$$\text{And clearly,} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Q7 :

Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Answer :

$x - y + 1 = 0$

$x = y - 1$

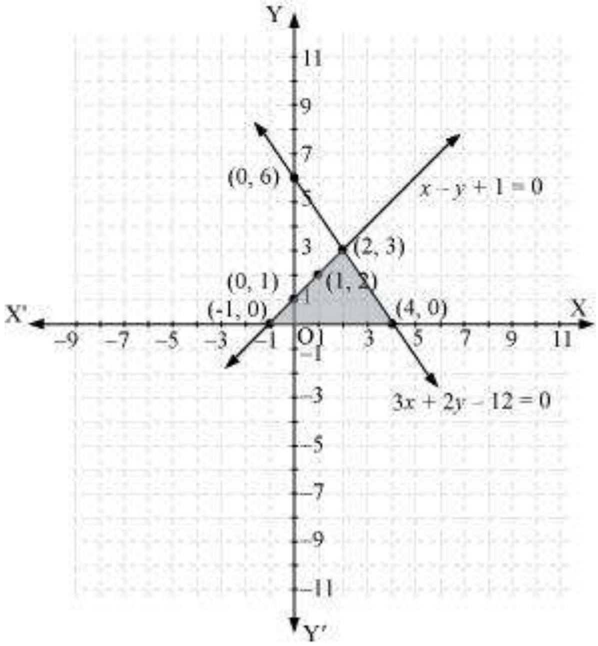
x	0	1	2
y	1	2	3

$3x + 2y - 12 = 0$

$$x = \frac{12 - 2y}{3}$$

x	4	2	0
y	0	3	6

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and *x*-axis at (- 1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), (- 1, 0), and (4, 0).

Exercise 3.3 : Solutions of Questions on Page Number : **53**

Q1 :

Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

(ii) $s - t = 3$

$\frac{s}{3} + \frac{t}{2} = 6$

(iii) $3x - y = 3$

$9x - 3y = 9$

(iv) $0.2x + 0.3y = 1.3$

$0.4x + 0.5y = 2.3$

(v) $\sqrt{2}x + \sqrt{3}y = 0$

$\sqrt{3}x - \sqrt{8}y = 0$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

Answer :

(i) *x* + *y* = 14 (1)

x - *y* = 4 (2)

From (1), we obtain

x = 14 - *y* (3)

Substituting this value in equation (2), we obtain

$$\begin{aligned}&(14-y)-y=4\\&14-2y=4\\&10=2y\\&y=5 \qquad (4)\end{aligned}$$

Substituting this in equation (3), we obtain

$$\begin{aligned}&x=9\\&\therefore x=9, y=5\end{aligned}$$

(ii) $s-t=3 \qquad (1)$

$$\frac{s}{3}+\frac{t}{2}=6 \qquad (2)$$

From (1), we obtain

$$s=t+3 \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\begin{aligned}&\frac{t+3}{3}+\frac{t}{2}=6\\&2t+6+3t=36\\&5t=30\\&t=6 \qquad (4)\end{aligned}$$

Substituting in equation (3), we obtain

$s = 9$

$\therefore s = 9, t = 6$

(iii)$3x - y = 3$ (1)

$9x - 3y = 9$ (2)

From (1), we obtain

$y = 3x - 3$ (3)

Substituting this value in equation (2), we obtain

$$\begin{aligned}&9x-3(3x-3)=9\\&9x-9x+9=9\end{aligned}$$

$9 = 9$

This is always true.

Hence, the given pair of equations has infinite possible solutions and the relation between these variables can be given by

$y = 3x - 3$

Therefore, one of its possible solutions is *x* = 1, *y* = 0.

(iv) $0.2x + 0.3y = 1.3 \qquad (1)$

$0.4x + 0.5y = 2.3 \qquad (2)$

From equation (1), we obtain

$$x = \frac{1.3 - 0.3y}{0.2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\begin{aligned} &0.4\left(\frac{1.3 - 0.3y}{0.2}\right) + 0.5y = 2.3 \\ &2.6 - 0.6y + 0.5y = 2.3 \\ &2.6 - 2.3 = 0.1y \\ &0.3 = 0.1y \\ &y = 3 \qquad (4) \end{aligned}$$

Substituting this value in equation (3), we obtain

$$\begin{aligned} &x = \frac{1.3 - 0.3 \times 3}{0.2} \\ &= \frac{1.3 - 0.9}{0.2} = \frac{0.4}{0.2} = 2 \\ &\therefore x = 2,\ y = 3 \end{aligned}$$

(v) $\sqrt{2}x + \sqrt{3}y = 0 \qquad (1)$

$\sqrt{3}x - \sqrt{8}y = 0 \qquad (2)$

From equation (1), we obtain

$$x = \frac{-\sqrt{3}y}{\sqrt{2}} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\begin{aligned} &\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0 \\ &-\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0 \end{aligned}$$

$$\begin{aligned} &y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0 \\ &y = 0 \qquad (4) \end{aligned}$$

Substituting this value in equation (3), we obtain

x

Q2 :

Solve 2*x* + 3*y* = 11 and 2*x* - 4*y* = - 24 and hence find the value of '*m*' for which *y* = *mx* + 3.

Answer :

$$\begin{aligned} 2x+3y&=11 && (1)\\ 2x-4y&=-24 && (2) \end{aligned}$$

From equation (1), we obtain

$$x=\frac{11-3y}{2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\begin{aligned} &2\left(\frac{11-3y}{2}\right)-4y=-24\\ &11-3y-4y=-24\\ &-7y=-35\\ &y=5 && (4) \end{aligned}$$

Putting this value in equation (3), we obtain

$$x=\frac{11-3\times5}{2}=-\frac{4}{2}=-2$$

Hence, *x* = - 2, *y* = 5

Also,

$$\begin{aligned} &y=mx+3\\ &5=-2m+3\\ &-2m=2\\ &m=-1 \end{aligned}$$

Q3 :

Solve 2*x* + 3*y* = 11 and 2*x* - 4*y* = - 24 and hence find the value of '*m*' for which *y* = *mx* + 3.

Answer :

$$2x+3y=11 \quad (1)$$
$$2x-4y=-24 \quad (2)$$

From equation (1), we obtain

$$x=\frac{11-3y}{2} \quad (3)$$

Substituting this value in equation (2), we obtain

$$2\left(\frac{11-3y}{2}\right)-4y=-24$$
$$11-3y-4y=-24$$
$$-7y=-35$$
$$y=5 \quad (4)$$

Putting this value in equation (3), we obtain

$$x=\frac{11-3\times 5}{2}=-\frac{4}{2}=-2$$

Hence, x = - 2, y = 5

Also,

$$y=mx+3$$
$$5=-2m+3$$
$$-2m=2$$
$$m=-1$$

Q4 :

Answer :

(i) Let the first number be x and the other number be y such that $y > x$.

According to the given information,

$$y=3x \quad (1)$$
$$y-x=26 \quad (2)$$

On substituting the value of y from equation (1) into equation (2), we obtain

$$3x-x=26$$
$$x=13 \quad (3)$$

Substituting this in equation (1), we obtain

y = 39

Hence, the numbers are 13 and 39.

(ii) Let the larger angle be x and smaller angle be y.

We know that the sum of the measures of angles of a supplementary pair is always 180 º.

According to the given information,

$$x + y = 180^\circ \qquad (1)$$
$$x - y = 18^\circ \qquad (2)$$

From (1), we obtain

$x = 180^\circ - y$ (3)

Substituting this in equation (2), we obtain

$$180^\circ - y - y = 18^\circ$$
$$162^\circ = 2y$$
$$81^\circ = y \qquad (4)$$

Putting this in equation (3), we obtain

$x = 180^\circ - 81^\circ$

$= 99^\circ$

Hence, the angles are 99 º and 81 º.

(iii) Let the cost of a bat and a ball be x and y respectively.

According to the given information,

$$7x + 6y = 3800 \qquad (1)$$
$$3x + 5y = 1750 \qquad (2)$$

From (1), we obtain

$$y = \frac{3800 - 7x}{6} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$$
$$3x + \frac{9500}{3} - \frac{35x}{6} = 1750$$
$$3x - \frac{35x}{6} = 1750 - \frac{9500}{3}$$

$$\frac{18x-35x}{6}=\frac{5250-9500}{3}$$
$$-\frac{17x}{6}=\frac{-4250}{3}$$
$$-17x=-8500$$
$$x=500 \qquad (4)$$

Substituting this in equation (3), we obtain

$$y=\frac{3800-7\times 500}{6}$$
$$=\frac{300}{6}=50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

(iv) Let the fixed charge be Rs *x* and per km charge be Rs *y*.

According to the given information,

$$x+10y=105 \qquad (1)$$
$$x+15y=155 \qquad (2)$$

From (1), we obtain

$$x=105-10y \qquad (3)$$

Substituting this in equation (2), we obtain

$$105-10y+15y=155$$
$$5y=50$$
$$y=10 \qquad (4)$$

Putting this in equation (3), we obtain

$$x=105-10\times 10$$
$$x=5$$

Hence, fixed charge = Rs 5

And per km charge = Rs 10

Charge for 25 km = $x + 25y$

= 5 + 250 = Rs 255

(v) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+2}{y+2}=\frac{9}{11}$$

$$11x+22=9y+18$$

$$11x-9y=-4 \qquad (1)$$

$$\frac{x+3}{y+3}=\frac{5}{6}$$

$$6x+18=5y+15$$

$$6x-5y=-3 \qquad (2)$$

From equation (1), we obtain

Exercise 3.4 : Solutions of Questions on Page Number : **56**

Q1 :

Solve the following pair of linear equations by the elimination method and the substitution method:

$(\text{i})\ \ x+y=5$ and $2x-3y=4$ $\ (\text{ii})\ \ 3x+4y=10$ and $2x-2y=2$

$(\text{iii})\ \ 3x-5y-4=0$ and $9x=2y+7$ $\ (\text{iv})\ \ \frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

Answer :

(i) ***By elimination method***

$$x+y=5 \qquad (1)$$

$$2x-3y=4 \qquad (2)$$

Multiplying equation (1) by 2, we obtain

$$2x+2y=10 \qquad (3)$$

Subtracting equation (2) from equation (3), we obtain

$$5y=6$$

$$y=\frac{6}{5} \qquad (4)$$

Substituting the value in equation (1), we obtain

$$x=5-\frac{6}{5}=\frac{19}{5}$$

$$\therefore x=\frac{19}{5}, y=\frac{6}{5}$$

By substitution method

From equation (1), we obtain

$x = 5 - y$ (5)

Putting this value in equation (2), we obtain

$$2(5-y)-3y=4$$

- 5y = - 6

$$y=\frac{6}{5}$$

Substituting the value in equation (5), we obtain

$$x=5-\frac{6}{5}=\frac{19}{5}$$
$$\therefore x=\frac{19}{5}, y=\frac{6}{5}$$

(ii) ***By elimination method***

$$3x+4y=10 \qquad (1)$$

$$2x-2y=2 \qquad (2)$$

Multiplying equation (2) by 2, we obtain

$$4x-4y=4 \qquad (3)$$

Adding equation (1) and (3), we obtain

$$7x=14$$
$$x=2 \qquad (4)$$

Substituting in equation (1), we obtain

$$6+4y=10$$
$$4y=4$$
$$y=1$$

Hence, x = 2, y = 1

By substitution method

From equation (2), we obtain

$x = 1 + y$ (5)

Putting this value in equation (1), we obtain

$$3(1+y)+4y=10$$

7y = 7

$y = 1$

Substituting the value in equation (5), we obtain

$$x = 1 + 1 = 2$$
$$\therefore x = 2, y = 1$$

(iii) ***By elimination method***

$$3x - 5y - 4 = 0 \qquad (1)$$

Q2 :

Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer :

(i)Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x+1}{y-1} = 1 \qquad \Rightarrow x - y = -2 \qquad (1)$$
$$\frac{x}{y+1} = \frac{1}{2} \qquad \Rightarrow 2x - y = 1 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$x = 3$ (3)

Substituting this value in equation (1), we obtain

$$\begin{aligned}3-y&=-2\\-y&=-5\\y&=5\end{aligned}$$

Hence, the fraction is $\frac{3}{5}$.

(ii)Let present age of Nuri = x

and present age of Sonu = y

According to the given information,

$$\begin{aligned}(x-5)&=3(y-5)\\x-3y&=-10 \qquad (1)\\(x+10)&=2(y+10)\\x-2y&=10 \qquad (2)\end{aligned}$$

Subtracting equation (1) from equation (2), we obtain

$y = 20$ (3)

Substituting it in equation (1), we obtain

$$\begin{aligned}x-60&=-10\\x&=50\end{aligned}$$

Hence, age of Nuri = 50 years

And, age of Sonu = 20 years

(iii)Let the unit digit and tens digits of the number be x and y respectively. Then, number = $10y + x$

Number after reversing the digits = $10x + y$

According to the given information,

$x + y = 9$ (1)

$9(10y + x) = 2(10x + y)$

$88y - 11x = 0$

$-x + 8y = 0$ (2)

Adding equation (1) and (2), we obtain

$9y = 9$

$y = 1$ (3)

Substituting the value in equation (1), we obtain

$x = 8$

Hence, the number is $10y + x = 10 \times 1 + 8 = 18$

(iv)Let the number of Rs 50 notes and Rs 100 notes be x and y respectively.

According to the given information,

$$x + y = 25 \qquad (1)$$
$$50x + 100y = 2000 \qquad (2)$$

Multiplying equation (1) by 50, we obtain

$$50x + 50y = 1250 \qquad (3)$$

Subtracting equation (3) from equation (2), we obtain

$$50y = 750$$
$$y = 15$$

Substituting in equation (1), we have *x* = 10

Hence, Meena has 10 notes of Rs 50 and 15 notes of Rs 100.

(v)Let the fixed charge for first three days and each day charge thereafter be Rs *x* and Rs *y* respectively.

According to the given information,

$$x + 4y = 27 \qquad (1)$$
$$x + 2y = 21 \qquad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$2y = 6$$
$$y = 3 \qquad (3)$$

Substituting in equation (1), we obtain

$$x + 12 = 27$$
$$x = 15$$

Hence, fixed charge = Rs 15

And Charge per day = Rs 3

Exercise 3.5 : Solutions of Questions on Page Number : **62**

Q1 :

Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

$$\begin{array}{llll} (\mathrm{i}) & x - 3y - 3 = 0 & (\mathrm{ii}) & 2x + y = 5 \\ & 3x - 9y - 2 = 0 & & 3x + 2y = 8 \\ (\mathrm{iii}) & 3x - 5y = 20 & (\mathrm{iv}) & x - 3y - 7 = 0 \\ & 6x - 10y = 40 & & 3x - 3y - 15 = 0 \end{array}$$

Answer :

$$\text{(i)}\quad x-3y-3=0$$

$$3x-9y-2=0$$

$$\frac{a_1}{a_2}=\frac{1}{3},\quad \frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\quad \frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

$$\text{(ii)}\quad 2x+y=5$$

$$3x+2y=8$$

$$\frac{a_1}{a_2}=\frac{2}{3},\quad \frac{b_1}{b_2}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{-5}{-8}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}$$

$$\frac{x}{-8-(-10)}=\frac{y}{-15+16}=\frac{1}{4-3}$$

$$\frac{x}{2}=\frac{y}{1}=1$$

$$\frac{x}{2}=1,\quad \frac{y}{1}=1$$

$$x=2,\quad y=1$$

∴ $x = 2$, $y = 1$

$$\text{(iii)}\quad 3x-5y=20$$

$$6x-10y=40$$

$$\frac{a_1}{a_2}=\frac{3}{6}=\frac{1}{2},\quad \frac{b_1}{b_2}=\frac{-5}{-10}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{-20}{-40}=\frac{1}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

$$\text{(iv)}\quad x-3y-7=0$$

$$3x-3y-15=0$$

$$\frac{a_1}{a_2}=\frac{1}{3},\quad \frac{b_1}{b_2}=\frac{-3}{-3}=1,\quad \frac{c_1}{c_2}=\frac{-7}{-15}=\frac{7}{15}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$$\frac{x}{45-(21)}=\frac{y}{-21-(-15)}=\frac{1}{-3-(-9)}$$

$$\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}$$

$$\frac{x}{24}=\frac{1}{6}\text{ and }\frac{y}{-6}=\frac{1}{6}$$

$$x=4\text{ and }y=-1$$

∴ $x=4, y=-1$

Q2 :

(i) For which values of *a* and *b* will the following pair of linear equations have an

infinite number of solutions?

$$2x+3y=7$$

$$(a-b)x+(a+b)y=3a+b-2$$

(ii) For which value of *k* will the following pair of linear equations have no solution?

$$3x+y=1$$

$$(2k-1)x+(k-1)y=2k+1$$

Answer :

$$\text{(i)}\quad 2x+3y-7=0$$

$$(a-b)x+(a+b)y-(3a+b-2)=0$$

$$\frac{a_1}{a_2}=\frac{2}{a-b},\quad \frac{b_1}{b_2}=\frac{3}{a+b},\quad \frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{7}{(3a+b-2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{7}{3a+b-2}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \qquad (1)$$

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \qquad (2)$$

Subtracting (1) from (2), we obtain

$$4b=4$$

$$b=1$$

Substituting this in equation (2), we obtain

$$a-5\times 1=0$$

$$a=5$$

Hence, *a* = 5 and *b* = 1 are the values for which the given equations give infinitely many solutions.

$$\text{(ii)} \qquad 3x+y-1=0$$

$$(2k-1)x+(k-1)y-2k-1=0$$

$$\frac{a_1}{a_2}=\frac{3}{2k-1}, \quad \frac{b_1}{b_2}=\frac{1}{k-1}, \quad \frac{c_1}{c_2}=\frac{-1}{-2k-1}=\frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$$\frac{3}{2k-1}=\frac{1}{k-1}\neq\frac{1}{2k+1}$$

$$\frac{3}{2k-1}=\frac{1}{k-1}$$

$$3k-3=2k-1$$

$$k=2$$

Hence, for *k* = 2, the given equation has no solution.

Q3 :

Solve the following pair of linear equations by the substitution and cross-multiplication methods:

$$8x+5y=9$$
$$3x+2y=4$$

Answer :

$$8x+5y=9 \qquad (i)$$
$$3x+2y=4 \qquad (ii)$$

From equation (*ii*), we obtain

$$x=\frac{4-2y}{3} \qquad (iii)$$

Substituting this value in equation (*i*), we obtain

$$8\left(\frac{4-2y}{3}\right)+5y=9$$
$$32-16y+15y=27$$
$$-y=-5$$
$$y=5 \qquad (iv)$$

Substituting this value in equation (*ii*), we obtain

$$3x+10=4$$
$$x=-2$$

Hence, $x=-2, y=5$

Again, by cross-multiplication method, we obtain

$$8x+5y-9=0$$
$$3x+2y-4=0$$
$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$
$$\frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$
$$\frac{x}{-2}=1 \text{ and } \frac{y}{5}=1$$
$$x=-2 \text{ and } y=5$$

Q4 :

Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i)A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii)A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

(iii)Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v)The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer :

(i)Let *x* be the fixed charge of the food and *y* be the charge for food per day.

According to the given information,

$$x+20y=1000 \qquad (1)$$
$$x+26y=1180 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$6y=180$$
$$y=30$$

Substituting this value in equation (1), we obtain

$$x+20\times30=1000$$
$$x=1000-600$$
$$x=400$$

Hence, fixed charge = Rs 400

And charge per day = Rs 30

(ii)Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x-1}{y}=\frac{1}{3} \quad \Rightarrow \quad 3x-y=3 \quad (1)$$
$$\frac{x}{y+8}=\frac{1}{4} \quad \Rightarrow \quad 4x-y=8 \quad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$x=5 \quad (3)$$

Putting this value in equation (1), we obtain

$$15-y=3$$
$$y=12$$

Hence, the fraction is $\frac{5}{12}$.

(iii)Let the number of right answers and wrong answers be *x* and *y*

respectively.

According to the given information,

$$3x-y=40 \quad (1)$$
$$4x-2y=50$$
$$\Rightarrow 2x-y=25 \quad (2)$$

Subtracting equation (2) from equation (1), we obtain

x = 15 (3)

Substituting this in equation (2), we obtain

$$30-y=25$$
$$y=5$$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv)Let the speed of 1st car and 2nd car be *u* km/h and *v* km/h.

Respective speed of both cars while they are travelling in same direction = ($u-v$) km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = ($u+v$) km/h

According to the given information,

$$5(u-v)=100$$
$$\Rightarrow u-v=20 \quad ...(1)$$
$$1(u+v)=100 \quad ...(2)$$

Adding both the equations, we obtain

$$2u = 120$$
$$u = 60 \text{ km/h} \qquad (3)$$

Substituting this value in equation (2), we obtain

v = 40 km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

(v) Let length and breadth of rectangle be *x* unit and *y* unit respectively.

Area = *xy*

According to the question,

$$(x-5)(y+3) = xy - 9$$
$$\Rightarrow 3x - 5y - 6 = 0 \qquad (1)$$
$$(x+3)(y+2) = xy + 67$$
$$\Rightarrow 2x + 3y - 61 = 0 \qquad (2)$$

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$
$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$
$$x = 17, y = 9$$

Exercise 3.6 : Solutions of Questions on Page Number : **67**

Q1 :

Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x} + \frac{1}{3y} = 2$

$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

$$(\text{v})\quad \frac{7x-2y}{xy}=5,\qquad \frac{8x+7y}{xy}=15$$

$$(\text{vi})\quad 6x+3y=6xy,\qquad 2x+4y=5xy$$

$$(\text{vii})\quad \frac{10}{x+y}+\frac{2}{x-y}=4,\qquad \frac{15}{x+y}-\frac{5}{x-y}=-2$$

$$(\text{viii})\quad \frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4},\qquad \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$$

Answer :

$$(\text{i})\quad \frac{1}{2x}+\frac{1}{3y}=2$$

$$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations change as follows.

$$\frac{p}{2}+\frac{q}{3}=2 \quad\Rightarrow\quad 3p+2q-12=0 \quad (1)$$

$$\frac{p}{3}+\frac{q}{2}=\frac{13}{6} \quad\Rightarrow\quad 2p+3q-13=0 \quad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\frac{p}{10}=\frac{1}{5}\text{ and }\frac{q}{15}=\frac{1}{5}$$

$$p=2\text{ and }q=3$$

$$\frac{1}{x}=2\text{ and }\frac{1}{y}=3$$

$$x=\frac{1}{2}\text{ and }y=\frac{1}{3}$$

$$(\text{ii})\qquad \frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Putting $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$ in the given equations, we obtain

$$2p+3q=2 \qquad (1)$$

$$4p-9q=-1 \qquad (2)$$

Multiplying equation (1) by 3, we obtain

$6p + 9q = 6$ (3)

Adding equation (2) and (3), we obtain

$$10p=5$$

$$p=\frac{1}{2} \qquad (4)$$

Putting in equation (1), we obtain

$$2\times\frac{1}{2}+3q=2$$

$$3q=1$$

$$q=\frac{1}{3}$$

$$p=\frac{1}{\sqrt{x}}=\frac{1}{2}$$

$$\sqrt{x}=2$$

$$x=4$$

$$\text{and } q=\frac{1}{\sqrt{y}}=\frac{1}{3}$$

$$\sqrt{y}=3$$

$$y=9$$

Hence, $x=4, y=9$

$$(\text{iii})\qquad \frac{4}{x}+3y=14$$

$$\frac{3}{x}-4y=23$$

Substituting $\frac{1}{x} = p$ in the given equations, we obtain

$$
\begin{aligned}
4p + 3y = 14 \quad &\Rightarrow \quad 4p + 3y - 14 = 0 \quad (1) \\
3p - 4y = 23 \quad &\Rightarrow \quad 3p - 4y - 23 = 0 \quad (2)
\end{aligned}
$$

By cross-multiplication, we obtain

$$
\begin{aligned}
&\frac{p}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9} \\
&\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25} \\
&\frac{p}{-125} = \frac{-1}{25} \text{ and } \frac{y}{50} = \frac{-1}{25} \\
&p = 5 \text{ and } y = -2 \\
&p = \frac{1}{x} = 5 \\
&\quad x = \frac{1}{5} \\
&\quad y = -2
\end{aligned}
$$

$$
\text{(iv)} \quad \frac{5}{x-1} + \frac{1}{y-2} = 2
$$

$$
\frac{6}{x-1} - \frac{3}{y-2} = 1
$$

Putting $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$ in the given equation, we obtain

$$
\begin{aligned}
5p + q = 2 \quad &(1) \\
6p - 3q = 1 \quad &(2)
\end{aligned}
$$

Multiplying equation (1) by 3, we obtain

Q2 :

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer :

(i)Let the speed of Ritu in still water and the speed of stream be *x* km/h

and *y* km/h respectively.

Speed of Ritu while rowing

Upstream = $(x-y)$ km/h

Downstream = $(x+y)$ km/h

According to question,

$$\begin{aligned} &2(x+y)=20 \\ &\Rightarrow x+y=10 \qquad (1) \\ &2(x-y)=4 \\ &\Rightarrow x-y=2 \qquad (2) \end{aligned}$$

Adding equation (1) and (2), we obtain

$$2x=12 \quad \Rightarrow \quad x=6$$

Putting this in equation (1), we obtain

$y = 4$

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii)Let the number of days taken by a woman and a man be *x* and *y* respectively.

Therefore, work done by a woman in 1 day = $\frac{1}{x}$

Work done by a man in 1 day = $\frac{1}{y}$

According to the question,

$$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$$

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$$

$$3\left(\frac{3}{x}+\frac{6}{y}\right)=1$$

$$\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$$

Putting $\frac{1}{x}=p \text{ and } \frac{1}{y}=q$ in these equations, we obtain

$$2p+5q=\frac{1}{4}$$

$$\Rightarrow \quad 8p+20q=1$$

$$3p+6q=\frac{1}{3}$$

$$\Rightarrow \; 9p+18q=1$$

By cross-multiplication, we obtain

$$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$$

$$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$$

$$\frac{p}{-2}=\frac{-1}{36} \text{ and } \frac{q}{-1}=\frac{1}{-36}$$

$$p=\frac{1}{18} \text{ and } q=\frac{1}{36}$$

$$p=\frac{1}{x}=\frac{1}{18} \text{ and } q=\frac{1}{y}=\frac{1}{36}$$

$$x=18 \qquad\qquad y=36$$

Hence, number of days taken by a woman = 18

Number of days taken by a man = 36

(iii) Let the speed of train and bus be *u* km/h and *v* km/h respectively.

According to the given information,

$$\frac{60}{u}+\frac{240}{v}=4 \qquad (1)$$

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \qquad (2)$$

Putting $\frac{1}{u}=p$ and $\frac{1}{v}=q$ in these equations, we obtain

$$60p+240q=4 \qquad (3)$$

$$100p+200q=\frac{25}{6}$$

$$600p+1200q=25 \qquad (4)$$

Multiplying equation (3) by 10, we obtain

$$600p+2400q=40 \qquad (5)$$

Subtracting equation (4) from (5), we obtain

$$1200q=15$$

$$q=\frac{15}{1200}=\frac{1}{80} \qquad (6)$$

Substituting in equation (3), we obtain

$$60p+3=4$$

$$60p=1$$

$$p=\frac{1}{60}$$

$$p=\frac{1}{u}=\frac{1}{60} \quad \text{and} \quad q=\frac{1}{v}=\frac{1}{80}$$

$$u=60\text{ km/h} \quad \text{and} \quad v=80\text{ km/h}$$

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h

Exercise 3.7 : Solutions of Questions on Page Number : **68**

Q1 :

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Answer :

The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years respectively.

Therefore, age of Ani's father, Dharam = 2 × x = $2x$ years

And age of Biju's sister Cathy $= \frac{y}{2}$ years

By using the information given in the question,

Case (I) When Ani is older than Biju by 3 years,

$x - y = 3$ (i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ (ii)

Subtracting (i) from (ii), we obtain

$3x = 60 - 3 = 57$

$$x = \frac{57}{3} = 19$$

Therefore, age of Ani = 19 years

And age of Biju = 19 - 3 = 16 years

Case (II) When Biju is older than Ani,

$y - x = 3$ (i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ (ii)

Adding (i) and (ii), we obtain

$3x = 63$

$x = 21$

Therefore, age of Ani = 21 years

And age of Biju = 21 + 3 = 24 years

Q2 :

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II)

[Hint: $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Answer :

Let those friends were having Rs x and y with them.

Using the information given in the question, we obtain

$x + 100 = 2(y - 100)$

$x + 100 = 2y - 200$

$x - 2y = -300$ (*i*)

And, $6(x - 10) = (y + 10)$

$6x - 60 = y + 10$

$6x - y = 70$ (*ii*)

Multiplying equation (*ii*) by 2, we obtain

$12x - 2y = 140$ (*iii*)

Subtracting equation (*i*) from equation (*iii*), we obtain

$11x = 140 + 300$

$11x = 440$

$x = 40$

Using this in equation (*i*), we obtain

$40 - 2y = -300$

$40 + 300 = 2y$

$2y = 340$

$y = 170$

Therefore, those friends had Rs 40 and Rs 170 with them respectively.

Q3 :

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer :

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km. We know that,

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$$

$$x = \frac{d}{t}$$

Or, $d = xt$ (*i*)

Using the information given in the question, we obtain

$$(x+10)=\frac{d}{(t-2)}$$

$$(x+10)(t-2)=d$$

$$xt+10t-2x-20=d$$

By using equation (*i*), we obtain

$-2x + 10t = 20$ (*ii*)

$$(x-10)=\frac{d}{(t+3)}$$

$$(x-10)(t+3)=d$$

$$xt-10t+3x-30=d$$

By using equation (*i*), we obtain

$3x - 10t = 30$ (*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$x = 50$

Using equation (*ii*), we obtain

$(-2) \times (50) + 10t = 20$

$-100 + 10t = 20$

$10t = 120$

$t = 12$ hours

From equation (*i*), we obtain

Distance to travel = $d = xt$

$= 50 \times 12$

$= 600$ km

Hence, the distance covered by the train is 600 km.

Q4 :

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer :

Let the number of rows be x and number of students in a row be y.

Total students of the class

= Number of rows x Number of students in a row

$= xy$

Using the information given in the question,

Condition 1

Total number of students = $(x - 1)(y + 3)$

$xy = (x - 1)(y + 3) = xy - y + 3x - 3$

$3x - y - 3 = 0$

$3x - y = 3$ (*i*)

Condition 2

Total number of students = $(x + 2)(y - 3)$

$xy = xy + 2y - 3x - 6$

$3x - 2y = -6$ (*ii*)

Subtracting equation (*ii*) from (*i*),

$(3x - y) - (3x - 2y) = 3 - (-6)$

$- y + 2y = 3 + 6$

$y = 9$

By using equation (*i*), we obtain

$3x - 9 = 3$

$3x = 9 + 3 = 12$

$x = 4$

Number of rows = $x = 4$

Number of students in a row = $y = 9$

Number of total students in a class = $xy = 4 \times 9 = 36$

Q5 :

In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Answer :

Given that,

$\angle C = 3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0$ … (*i*)

We know that the sum of the measures of all angles of a triangle is 180°. Therefore,

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 3\angle B = 180°$

$\angle A + 4\angle B = 180°$ … (*ii*)

Multiplying equation (*i*) by 4, we obtain

$8\angle A - 4\angle B = 0$ … (*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$9\angle A = 180°$

$\angle A = 20°$

From equation (*ii*), we obtain

$20° + 4\angle B = 180°$

$4\angle B = 160°$

$\angle B = 40°$

$\angle C = 3\angle B$

$= 3 \times 40° = 120°$

Therefore, ∠ A, ∠ B, ∠ C are 20°, 40°, and 120° respectively.

Q6 :

Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the *y* axis.

Answer :

$5x - y = 5$

Or, $y = 5x - 5$

The solution table will be as follows.

x	0	1	2
y	- 5	0	5

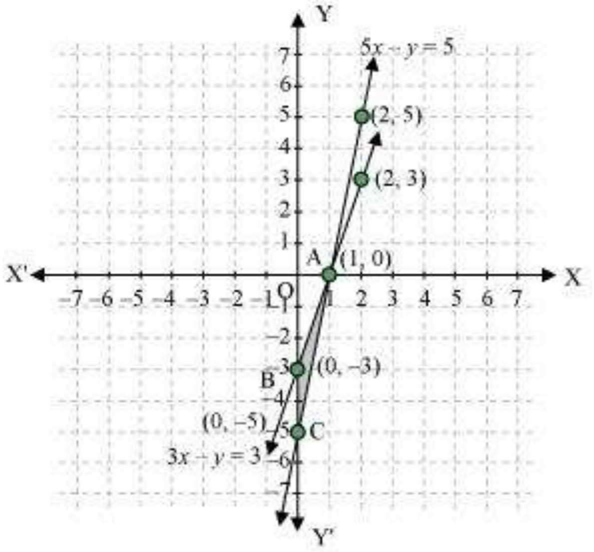
$3x - y = 3$

Or, $y = 3x - 3$

The solution table will be as follows.

x	0	1	2
y	- 3	0	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ΔABC formed by these lines and *y*-axis.

The coordinates of vertices are A (1, 0), B (0, - 3), C (0, - 5).

Q7 :

Solve the following pair of linear equations.

(i) *px* + *qy* = *p* - *q*

qx* - *py* = *p* + *q

(ii) *ax* + *by* = *c*

bx* + *ay* = 1 + *c

(iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

(iv) $(a - b)\, x + (a + b)\, y = a^2 - 2ab - b^2$

$(a + b)\,(x + y) = a^2 + b^2$

(v) 152*x* - 378*y* = - 74

- 378*x* + 152*y* = - 604

Answer :

(i)*px* + *qy* = *p* - *q* … (1)

qx - *py* = *p* + *q* … (2)

Multiplying equation (1) by *p* and equation (2) by *q*, we obtain

$p^2x + pqy = p^2 - pq$ … (3)

$q^2x - pqy = pq + q^2$ … (4)

Adding equations (3) and (4), we obtain

$p^2x + q^2 x = p^2 + q^2$

$(p^2 + q^2) x = p^2 + q^2$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

From equation (1), we obtain

$p (1) + qy = p - q$

$qy = - q$

$y = - 1$

(ii)$ax + by = c$ … (1)

$bx + ay = 1 + c$ … (2)

Multiplying equation (1) by a and equation (2) by b, we obtain

$a^2x + aby = ac$ … (3)

$b^2x + aby = b + bc$ … (4)

Subtracting equation (4) from equation (3),

$(a^2 - b^2) x = ac - bc - b$

$$x = \frac{c(a-b)-b}{a^2-b^2}$$

From equation (1), we obtain

$ax + by = c$

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\} + by = c$$

$$\frac{ac(a-b)-ab}{a^2-b^2} + by = c$$

$$by = c - \frac{ac(a-b)-ab}{a^2-b^2}$$

$$by = \frac{a^2c - b^2c - a^2c + abc + ab}{a^2-b^2}$$

$$by = \frac{abc - b^2c + ab}{a^2-b^2}$$

$$by = \frac{bc(a-b)+ab}{a^2-b^2}$$

$$y = \frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$

Or, $bx - ay = 0$ … (1)

$ax + by = a^2 + b^2$ … (2)

Multiplying equation (1) and (2) by b and a respectively, we obtain

$b^2x - aby = 0$ … (3)

$a^2x + aby = a^3 + ab^2$ … (4)

Adding equations (3) and (4), we obtain

$b^2x + a^2x = a^3 + ab^2$

$x (b^2 + a^2) = a (a^2 + b^2)$

$x = a$

By using (1), we obtain

$b (a) - ay = 0$

$ab - ay = 0$

$ay = ab$

$y = b$

(iv) $(a - b) x + (a + b) y = a^2 - 2ab - b^2$ … (1)

$(a + b) (x + y) = a^2 + b^2$

$(a + b) x + (a + b) y = a^2 + b^2$ … (2)

Subtracting equation (2) from (1), we obtain

$(a - b) x - (a + b) x = (a^2 - 2ab - b^2) - (a^2 + b^2)$

$(a - b - a - b) x = - 2ab - 2b^2$

$- 2bx = - 2b (a + b)$

$x = a + b$

Using equation (1), we obtain

$(a - b) (a + b) + (a + b) y = a^2 - 2ab - b^2$

$a^2 - b^2 + (a + b) y = a^2 - 2ab - b^2$

$(a + b) y = - 2ab$

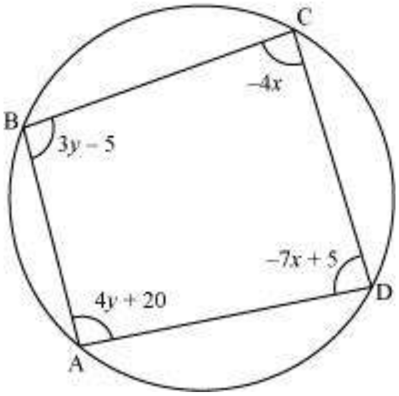
$$y=\frac{-2ab}{a+b}$$

(v) $152x - 378y = - 74$

$76x - 189y = - 37$

Q8 :

ABCD is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Answer :

We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.

Therefore, ∠ A + ∠ C = 180

$4y + 20 - 4x = 180$

$- 4x + 4y = 160$

$x - y = - 40$ (*i*)

Also, ∠ B + ∠ D = 180

$3y - 5 - 7x + 5 = 180$

$- 7x + 3y = 180$ (*ii*)

Multiplying equation (*i*) by 3, we obtain

$3x - 3y = - 120$ (*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$- 7x + 3x = 180 - 120$

$- 4x = 60$

$x = -15$

By using equation (*i*), we obtain

$x - y = - 40$

$-15 - y = - 40$

$y = -15 + 40 = 25$

∠ A = $4y + 20 = 4(25) + 20 = 120°$

∠ B = $3y - 5 = 3(25) - 5 = 70°$

∠ C = $- 4x = - 4(- 15) = 60°$

$\angle D = -7x + 5 = -7(-15) + 5 = 110°$

DELHI PUBLIC SCHOOL, GANDHINAGAR

CH.5 ARITHMETIC PROGRESSIONS

MIND MAP

This chapter consists of two different topics. The most probable questions from the examination point of view are given below.

TYPE: 1 TO FIND THE n^{th} TERM OF AN A.P.

1. Write fourth term of an A.P. if its nth term is 3n+2.
2. Find whether 0 is a term of the A.P: 40,37,34,31, . .
3. Write the next four terms of an A.P. $\sqrt{2}$, $\sqrt{18}$…
4. If $\frac{4}{5}, a, 2,$ …. three consecutive term of an A.P then find 'a'.
5. Find the middle term of A.P. 6,13,20,………..216.
6. Find A.P which fifth term is 5 and common difference is –3.
7. Determine the 15th from the end of the A.P: 4,9,14……..254 .
8. The nth term of an A.P is 3n+5 find its common difference.

TYPE: 2 TO FIND THE SUM OF n^{th} TERM OF AN A.P.

1. The sum of first 6 term of A.P is 42.The ratio its 10th term to 30th term is1:3. Calculate the first and 13th term of the A.P.
2. Find the sum of first 24 term of A.P. 5, 8, 11, 14….
3. The sum of three numbers in A.P is 21and their product is 231. Find the numbers.
4. Find the sum of 25 terms of an A.P. whose nth term is given by $7 - 3n$.
5. Find the sum of all two digit odd positive numbers.
6. Find the sum of three digits numbers that are divisible by 17.
7. How many term of the A.P: 17, 15, 13…must be added to get the sum 72? Explain the double answer.
8. The sums of $n, 2n, 3n$ term of an A.P are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

NCERT Solutions for Class 10 Maths Unit 5

Arithmetic Progressions Class 10

Unit 5 Arithmetic Progressions Exercise 5.1, 5.2, 5.3, 5.4 Solutions

Exercise 5.1 : Solutions of Questions on Page Number : **99**

Q1 :

In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv)The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Answer :

(i) It can be observed that

Taxi fare for 1^{st} km = 15

Taxi fare for first 2 km = 15 + 8 = 23

Taxi fare for first 3 km = 23 + 8 = 31

Taxi fare for first 4 km = 31 + 8 = 39

Clearly 15, 23, 31, 39 … forms an A.P. because every term is 8 more than the preceding term.

(ii) Let the initial volume of air in a cylinder be *V* lit. In each stroke, the vacuum pump removes $\frac{1}{4}$ of air remaining in the cylinder at a time. In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}\text{th}$ part of air will remain.

Therefore, volumes will be $V, \frac{3}{4}V, \left(\frac{3}{4}\right)^2 V, \left(\frac{3}{4}\right)^3 V \ldots\ldots\ldots$

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii) Cost of digging for first metre = 150

Cost of digging for first 2 metres = 150 + 50 = 200

Cost of digging for first 3 metres = 200 + 50 = 250

Cost of digging for first 4 metres = 250 + 50 = 300

Clearly, 150, 200, 250, 300 … forms an A.P. because every term is 50 more than the preceding term.

(iv) We know that if Rs P is deposited at *r*% compound interest per annum for *n* years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after*n* years.

Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right),10000\left(1+\frac{8}{100}\right)^2,10000\left(1+\frac{8}{100}\right)^3,10000\left(1+\frac{8}{100}\right)^4,\ldots$$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Q2 :

Write first four terms of the A.P. when the first term *a* and the common difference*d* are given as follows

(i) *a* = 10, *d* = 10

(ii) *a* = - 2, *d* = 0

(iii) *a* = 4,*d* = - 3

(iv) *a* = - 1 *d* = $\frac{1}{2}$

(v) *a* = - 1.25, *d* = - 0.25

Answer :

(i) *a* = 10, *d* = 10

Let the series be a_1, a_2, a_3, a_4, a_5 …

$a_1 = a = 10$

$a_2 = a_1 + d = 10 + 10 = 20$

$a_3 = a_2 + d = 20 + 10 = 30$

$a_4 = a_3 + d = 30 + 10 = 40$

$a_5 = a_4 + d = 40 + 10 = 50$

Therefore, the series will be 10, 20, 30, 40, 50 …

First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) *a* = - 2, *d* = 0

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a = - 2$

$a_2 = a_1 + d = - 2 + 0 = - 2$

$a_3 = a_2 + d = - 2 + 0 = - 2$

$a_4 = a_3 + d$ = - 2 + 0 = - 2

Therefore, the series will be - 2, - 2, - 2, - 2 …

First four terms of this A.P. will be - 2, - 2, - 2 and - 2.

(iii) a = 4,d = - 3

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a$ = 4

$a_2 = a_1 + d$ = 4 - 3 = 1

$a_3 = a_2 + d$ = 1 - 3 = - 2

$a_4 = a_3 + d$ = - 2 - 3 = - 5

Therefore, the series will be 4, 1, - 2 - 5 …

First four terms of this A.P. will be 4, 1, - 2 and - 5.

(iv) a = - 1, d = $\frac{1}{2}$

Let the series be a_1, a_2, a_3, a_4 …

$$a_1 = a = -1$$
$$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$$
$$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$$
$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Clearly, the series will be

$-1, -\frac{1}{2}, 0, \frac{1}{2}$ ………….

First four terms of this A.P. will be $-1, -\frac{1}{2}, 0 \text{ and } \frac{1}{2}$.

(v) a = - 1.25, d = - 0.25

Let the series be a_1, a_2, a_3, a_4 …

$a_1 = a$ = - 1.25

$a_2 = a_1 + d$ = - 1.25 - 0.25 = - 1.50

$a_3 = a_2 + d$ = - 1.50 - 0.25 = - 1.75

$a_4 = a_3 + d$ = - 1.75 - 0.25 = - 2.00

Clearly, the series will be 1.25, - 1.50, - 1.75, - 2.00 ……..

First four terms of this A.P. will be - 1.25, - 1.50, - 1.75 and - 2.00.

Q3 :

For the following A.P.s, write the first term and the common difference.

(i) 3, 1, - 1, - 3 …

(ii) - 5, - 1, 3, 7 …

(iii) $\frac{1}{3},\frac{5}{3},\frac{9}{3},\frac{13}{3}\ldots$

(iv) 0.6, 1.7, 2.8, 3.9 …

Answer :

(i) 3, 1, - 1, - 3 …

Here, first term, *a* = 3

Common difference, *d* = Second term - First term

= 1 - 3 = - 2

(ii) - 5, - 1, 3, 7 …

Here, first term, *a* = - 5

Common difference, *d* = Second term - First term

= (- 1) - (- 5) = - 1 + 5 = 4

(iii) $\frac{1}{3},\frac{5}{3},\frac{9}{3},\frac{13}{3}\ldots$

Here, first term, $a=\frac{1}{3}$

Common difference, *d* = Second term - First term

$=\frac{5}{3}-\frac{1}{3}=\frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 …

Here, first term, *a* = 0.6

Common difference, *d* = Second term - First term

= 1.7 - 0.6

= 1.1

Q4 :

Which of the following are APs? If they form an A.P. find the common difference *d* and write three more terms.

(i) 2, 4, 8, 16 …

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

(iii) - 1.2, - 3.2, - 5.2, - 7.2 …

(iv) - 10, - 6, - 2, 2 …

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2} \ldots$

(vi) 0.2, 0.22, 0.222, 0.2222 ….

(vii) 0, - 4, - 8, - 12 …

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2} \ldots$

(ix) 1, 3, 9, 27 …

(x) $a, 2a, 3a, 4a$ **…**

(xi) a, a^2, a^3, a^4 **…**

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32} \ldots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12} \ldots$

(xiv) $1^2, 3^2, 5^2, 7^2$ **…**

(xv) $1^2, 5^2, 7^2, 73$ **…**

Answer :

(i) 2, 4, 8, 16 …

It can be observed that

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

i.e., $a_{k+1} - a_k$ is not the same every time. Therefore, the given numbers are not forming an A.P.

(ii) $2, \frac{5}{2}, 3, \frac{7}{2} \ldots$

It can be observed that

$$a_2 - a = \frac{5}{2} - 2 = \frac{1}{2}$$
$$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$$
$$a_4 - a_3 = \frac{7}{2} - 3 = \frac{1}{2}$$

i.e., a_{k+1} - a_k is same every time.

Therefore, $d = \frac{1}{2}$ and the given numbers are in A.P.

Three more terms are

$$a_5 = \frac{7}{2} + \frac{1}{2} = 4$$
$$a_6 = 4 + \frac{1}{2} = \frac{9}{2}$$
$$a_7 = \frac{9}{2} + \frac{1}{2} = 5$$

(iii) - 1.2, - 3.2, - 5.2, - 7.2 …

It can be observed that

a_2 - a_1 = (- 3.2) - (- 1.2) = - 2

a_3 - a_2 = (- 5.2) - (- 3.2) = - 2

a_4 - a_3 = (- 7.2) - (- 5.2) = - 2

i.e., a_{k+1} - a_k is same every time. Therefore, d = - 2

The given numbers are in A.P.

Three more terms are

a_5 = - 7.2 - 2 = - 9.2

a_6 = - 9.2 - 2 = - 11.2

a_7 = - 11.2 - 2 = - 13.2

(iv) - 10, - 6, - 2, 2 …

It can be observed that

a_2 - a_1 = (- 6) - (- 10) = 4

a_3 - a_2 = (- 2) - (- 6) = 4

a_4 - a_3 = (2) - (- 2) = 4

i.e., a_{k+1} - a_k is same every time. Therefore, d = 4

The given numbers are in A.P.

Three more terms are

$a_5 = 2 + 4 = 6$

$a_6 = 6 + 4 = 10$

$a_7 = 10 + 4 = 14$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

It can be observed that

$$a_2 - a_1 = 3+\sqrt{2}-3 = \sqrt{2}$$
$$a_3 - a_2 = 3+2\sqrt{2}-3-\sqrt{2} = \sqrt{2}$$
$$a_4 - a_3 = 3+3\sqrt{2}-3-2\sqrt{2} = \sqrt{2}$$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = \sqrt{2}$

The given numbers are in A.P.

Three more terms are

$$a_5 = 3+3\sqrt{2}+\sqrt{2} = 3+4\sqrt{2}$$
$$a_6 = 3+4\sqrt{2}+\sqrt{2} = 3+5\sqrt{2}$$
$$a_7 = 3+5\sqrt{2}+\sqrt{2} = 3+6\sqrt{2}$$

(vi) 0.2, 0.22, 0.222, 0.2222 ….

It can be observed that

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

i.e., $a_{k+1} - a_k$ is not the same every time.

Therefore, the given numbers are not in A.P.

(vii) 0, - 4, - 8, - 12 …

It can be observed that

$a_2 - a_1 = (-4) - 0 = -4$

$a_3 - a_2 = (-8) - (-4) = -4$

$a_4 - a_3 = (-12) - (-8) = -4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, $d = -4$

The given numbers are in A.P.

Three more terms are

$a_5 = -$

Exercise 5.2 : Solutions of Questions on Page Number : **105**

Q1 :

Fill in the blanks in the following table, given that *a* is the first term, *d* the common difference and a_n the n^{th}term of the A.P.

	a	d	n	a_n
I	7	3	8	…...
II	- 18	…..	10	0
III	…..	- 3	18	- 5
IV	- 18.9	2.5	…..	3.6
V	3.5	0	105	…..

Answer :

I. $a = 7$, $d = 3$, $n = 8$, $a_n = ?$

We know that,

For an A.P. $a_n = a + (n - 1) d$

$= 7 + (8 - 1) 3$

$= 7 + (7) 3$

$= 7 + 21 = 28$

Hence,$a_n = 28$

II. Given that

$a = - 18$, $n = 10$, $a_n = 0$, $d = ?$

We know that,

$a_n = a + (n - 1) d$

$0 = - 18 + (10 - 1) d$

$18 = 9d$

$$d = \frac{18}{9} = 2$$

Hence, common difference, $d = 2$

III. Given that

$d = - 3$, $n = 18$, $a_n = - 5$

We know that,

$a_n = a + (n - 1) d$

$- 5 = a + (18 - 1) (- 3)$

$- 5 = a + (17) (- 3)$

$-5 = a - 51$

$a = 51 - 5 = 46$

Hence, $a = 46$

IV. $a = -18.9$, $d = 2.5$, $a_n = 3.6$, $n = ?$

We know that,

$a_n = a + (n - 1)d$

$3.6 = -18.9 + (n - 1)2.5$

$3.6 + 18.9 = (n - 1)2.5$

$22.5 = (n - 1)2.5$

$$(n-1)=\frac{22.5}{2.5}$$

$$n-1=9$$

$$n=10$$

Hence, $n = 10$

V. $a = 3.5$, $d = 0$, $n = 105$, $a_n = ?$

We know that,

$a_n = a + (n - 1)d$

$a_n = 3.5 + (105 - 1)0$

$a_n = 3.5 + 104 \times 0$

$a_n = 3.5$

Hence, $a_n = 3.5$

Q2 :

Choose the correct choice in the following and justify

I. 30th term of the A.P: 10, 7, 4, …, is

A. 97 B. 77 C. - 77 D. - 87

II 11th term of the A.P. $-3, -\frac{1}{2}, 2, \ldots$ **is**

A. 28 B. 22 C. - 38 D. $-48\frac{1}{2}$

Answer :

I. Given that

A.P. 10, 7, 4, …

First term, $a = 10$

Common difference, $d = a_2 - a_1 = 7 - 10$

$= - 3$

We know that, $a_n = a + (n - 1) d$

$a_{30} = 10 + (30 - 1) (- 3)$

$a_{30} = 10 + (29) (- 3)$

$a_{30} = 10 - 87 = - 77$

Hence, the correct answer is **C**.

II. Given that, A.P. $-3, -\frac{1}{2}, 2, \ldots$

First term $a = - 3$

Common difference, $d = a_2 - a_1$

$$= -\frac{1}{2} - (-3)$$

$$= -\frac{1}{2} + 3 = \frac{5}{2}$$

We know that,

$$a_n = a + (n-1)d$$

$$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$$

$$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$$

$$a_{11} = -3 + 25$$

$$a_{11} = 22$$

Hence, the answer is **B**.

Q3 :

In the following APs find the missing term in the boxes

I. $2, \square, 26$

II. $\square, 13, \square, 3$

III. $5, \square, \square, 9\frac{1}{2}$

IV. $-4, \square, \square, \square, \square, 6$

V. $\square, 38, \square, \square, \square, -22$

Answer :

I. $2, \square, 26$

For this A.P.,

$a = 2$

$a_3 = 26$

We know that, $a_n = a + (n - 1) d$

$a_3 = 2 + (3 - 1) d$

$26 = 2 + 2d$

$24 = 2d$

$d = 12$

$a_2 = 2 + (2 - 1) 12$

$= 14$

Therefore, 14 is the missing term.

II. $\square, 13, \square, 3$

For this A.P.,

$a_2 = 13$ and

$a_4 = 3$

We know that, $a_n = a + (n - 1) d$

$a_2 = a + (2 - 1) d$

$13 = a + d$ (I)

$a_4 = a + (4 - 1) d$

$3 = a + 3d$ (II)

On subtracting (I) from (II), we obtain

$- 10 = 2d$

$d = - 5$

From equation (I), we obtain

$13 = a + (- 5)$

$a = 18$

$a_3 = 18 + (3 - 1) (- 5)$

$= 18 + 2 (- 5) = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

III. $5, \square, \square, 9\frac{1}{2}$

For this A.P.,

$$a = 5$$

$$a_4 = 9\frac{1}{2} = \frac{19}{2}$$

We know that,

$$a_n = a + (n-1)d$$

$$a_4 = a + (4-1)d$$

$$\frac{19}{2} = 5 + 3d$$

$$\frac{19}{2} - 5 = 3d$$

$$\frac{9}{2} = 3d$$

$$d = \frac{3}{2}$$

$$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$$

$$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$$

Therefore, the missing terms are $\frac{13}{2}$ and 8 respectively.

IV. $-4, \square, \square, \square, \square, 6$

For this A.P.,

a = - 4 and

a_6 = 6

We know that,

$a_n = a + (n - 1)\, d$

$a_6 = a + (6 - 1)\, d$

$6 = - 4 + 5d$

$10 = 5d$

$d = 2$

$a_2 = a + d = - 4 + 2 = - 2$

$a_3 = a + 2d = -4 + 2(2) = 0$

$a_4 = a + 3d = -4 + 3(2) = 2$

$a_5 = a + 4d = -4 + 4(2) = 4$

Therefore, the missing terms are - 2, 0, 2, and 4 respectively.

V. $\square, 38, \square, \square, \square, -22$

For this A.P.,

$a_2 = 38$

$a_6 = -22$

We know that

$a_n = a + (n - 1)\,d$

$a_2 = a + (2 - 1)\,d$

$38 = a + d$ (1)

$a_6 = a + (6 - 1)\,d$

$-22 = a + 5d$ (2)

On subtracting equation (1) from (2), we obtain

$-22 - 38 = 4d$

$-60 = 4d$

$d = -15$

$a = a_2 - d = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2(-15) = 23$

$a_4 = a + 3d = 53 + 3(-15) = 8$

$a_5 = a + 4d = 53 + 4(-15) = -7$

Therefore, the missing terms are 53, 23, 8, and - 7 respectively.

Q4 :

Which term of the A.P. 3, 8, 13, 18, … is 78?

Answer :

3, 8, 13, 18, …

For this A.P.,

$a = 3$

$d = a_2 - a_1 = 8 - 3 = 5$

Let n^{th} term of this A.P. be 78.

$a_n = a + (n - 1)\ d$

$78 = 3 + (n - 1)\ 5$

$75 = (n - 1)\ 5$

$(n - 1) = 15$

$n = 16$

Hence, 16th term of this A.P. is 78.

Q5 :

Find the number of terms in each of the following A.P.

I. 7, 13, 19, …, 205

II. $18, 15\frac{1}{2}, 13, \ldots, -47$

Answer :

I. 7, 13, 19, …, 205

For this A.P.,

$a = 7$

$d = a_2 - a_1 = 13 - 7 = 6$

Let there are n terms in this A.P.

$a_n = 205$

We know that

$a_n = a + (n - 1)\ d$

Therefore, $205 = 7 + (n - 1)\ 6$

$198 = (n - 1)\ 6$

$33 = (n - 1)$

$n = 34$

Therefore, this given series has 34 terms in it.

II. $18,\ 15\frac{1}{2}, 13, \ldots, -47$

For this A.P.,

$$a = 18$$

$$d = a_2 - a_1 = 15\frac{1}{2} - 18$$

$$d = \frac{31-36}{2} = -\frac{5}{2}$$

Let there are *n* terms in this A.P.

Therefore, $a_n = -47$ and we know that,

$$a_n = a + (n-1)d$$

$$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$

$$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$$

$$-65 = (n-1)\left(-\frac{5}{2}\right)$$

$$(n-1) = \frac{-130}{-5}$$

$$(n-1) = 26$$

$$n = 27$$

Therefore, this given A.P. has 27 terms in it.

Q6 :

Check whether - 150 is a term of the A.P. 11, 8, 5, 2, …

Answer :

For this A.P.,

$a = 11$

$d = a_2 - a_1 = 8 - 11 = -3$

Let - 150 be the n^{th} term of this A.P.

We know that,

$$a_n = a + (n-1)d$$

$$-150 = 11 + (n-1)(-3)$$

$$-150 = 11 - 3n + 3$$

$$-164 = -3n$$

$$n = \frac{164}{3}$$

Clearly, n is not an integer.

Therefore, - 150 is not a term of this A.P.

Q7 :

Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73

Answer :

Given that,

a_{11} = 38

a_{16} = 73

We know that,

$a_n = a + (n - 1) d$

$a_{11} = a + (11 - 1) d$

$38 = a + 10d$ (1)

Similarly,

$a_{16} = a + (16 - 1) d$

$73 = a + 15d$ (2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

38 = a + 10 x (7)

38 - 70 = a

a = -32

$a_{31} = a + (31 - 1) d$

= - 32 + 30 (7)

= - 32 + 210

= 178

Hence, 31st term is 178.

Q8 :

An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term

Answer :

Given that,

$a_3 = 12$

$a_{50} = 106$

We know that,

$a_n = a + (n - 1) d$

$a_3 = a + (3 - 1) d$

$12 = a + 2d$ (I)

Similarly, $a_{50} = a + (50 - 1) d$

$106 = a + 49d$ (II)

On subtracting (I) from (II), we obtain

$94 = 47d$

$d = 2$

From equation (I), we obtain

$12 = a + 2 (2)$

$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1) d$

$a_{29} = 8 + (28)2$

$a_{29} = 8 + 56 = 64$

Therefore, 29th term is 64.

Q9 :

If the 3rd and the 9th terms of an A.P. are 4 and - 8 respectively. Which term of this A.P. is zero.

Answer :

Given that,

$a_3 = 4$

$a_9 = -8$

We know that,

$a_n = a + (n - 1) d$

$a_3 = a + (3 - 1) d$

$4 = a + 2d$ (I)

$a_9 = a + (9 - 1) d$

$-8 = a + 8d$ (II)

On subtracting equation (I) from (II), we obtain

$-12 = 6d$

$d = -2$

From equation (I), we obtain

$4 = a + 2 (-2)$

$4 = a - 4$

$a = 8$

Let n^{th} term of this A.P. be zero.

$a_n = a + (n - 1) d$

$0 = 8 + (n - 1) (-2)$

$0 = 8 - 2n + 2$

$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

Q10 :

If 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Answer :

We know that,

For an A.P., $a_n = a + (n - 1) d$

$a_{17} = a + (17 - 1) d$

$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$

It is given that

$a_{17} - a_{10} = 7$

$(a + 16d) - (a + 9d) = 7$

$7d = 7$

$d = 1$

Therefore, the common difference is 1.

Q11 :

Which term of the A.P. 3, 15, 27, 39, … will be 132 more than its 54th term?

Answer :

Given A.P. is 3, 15, 27, 39, …

$a = 3$

$d = a_2 - a_1 = 15 - 3 = 12$

$a_{54} = a + (54 - 1)\, d$

$= 3 + (53)\,(12)$

$= 3 + 636 = 639$

$132 + 639 = 771$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.

$a_n = a + (n - 1)\, d$

$771 = 3 + (n - 1)\, 12$

$768 = (n - 1)\, 12$

$(n - 1) = 64$

$n = 65$

Therefore, 65th term was 132 more than 54th term.

Alternatively,

Let n^{th} term be 132 more than 54th term.

$$n = 54 + \frac{132}{12}$$

$$= 54 + 11 = 65^{\text{th}} \text{ term}$$

Q12 :

Two APs have the same common difference. The difference between their 100th term is 100, what is the difference between their 1000th terms?

Answer :

Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d.

For first A.P.,

$a_{100} = a_1 + (100 - 1)\, d$

$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)\, d$

$a_{1000} = a_1 + 999d$

For second A.P.,

$a_{100} = a_2 + (100 - 1)\, d$

$= a_2 + 99d$

$a_{1000} = a_2 + (1000 - 1)\, d$

$= a_2 + 999d$

Given that, difference between

100th term of these A.P.s = 100

Therefore, $(a_1 + 99d) - (a_2 + 99d) = 100$

$a_1 - a_2 = 100$ (1)

Difference between 1000th terms of these A.P.s

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$

From equation (1),

This difference, $a_1 - a_2 = 100$

Hence, the difference between 1000th terms of these A.P. will be 100.

Q13 :

How many three digit numbers are divisible by 7

Answer :

First three-digit number that is divisible by 7 = 105

Next number = 105 + 7 = 112

Therefore, 105, 112, 119, …

All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, 999 - 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.

105, 112, 119, …, 994

Let 994 be the *n*th term of this A.P.

$a = 105$

$d = 7$

$a_n = 994$

$n = ?$

$a_n = a + (n - 1) d$

$994 = 105 + (n - 1) 7$

$889 = (n - 1) 7$

$(n - 1) = 127$

$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Q14 :

How many multiples of 4 lie between 10 and 250?

Answer :

First multiple of 4 that is greater than 10 is 12. Next will be 16.

Therefore, 12, 16, 20, 24, …

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2. Therefore, 250 - 2 = 248 is divisible by 4.

The series is as follows.

12, 16, 20, 24, …, 248

Let 248 be the n^{th} term of this A.P.

$$a = 12$$

$$d = 4$$

$$a_n = 248$$

$$a_n = a + (n-1)d$$

$$248 = 12 + (n-1)4$$

$$\frac{236}{4} = n-1$$

$$59 = n-1$$

$$n = 60$$

Therefore, there are 60 multiples of 4 between 10 and 250.

Q15 :

For what value of n, are the n^{th} terms of two APs 63, 65, 67, and 3, 10, 17, … equal

Answer :

63, 65, 67, …

$a = 63$

$d = a_2 - a_1 = 65 - 63 = 2$

n^{th} term of this A.P. $= a_n = a + (n - 1) d$

$a_n= 63 + (n - 1) 2 = 63 + 2n - 2$

$a_n = 61 + 2n$ (1)

3, 10, 17, …

$a = 3$

$d = a_2 - a_1 = 10 - 3 = 7$

n^{th} term of this A.P. $= 3 + (n - 1) 7$

$a_n= 3 + 7n - 7$

$a_n = 7n - 4$ (2)

It is given that, n^{th} term of these A.P.s are equal to each other.

Equating both these equations, we obtain

$61 + 2n = 7n - 4$

$61 + 4 = 5n$

$5n = 65$

$n = 13$

Therefore, 13^{th} terms of both these A.P.s are equal to each other.

Q16 :

Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Answer :

$=a_3 = 16$

$a + (3 - 1) d = 16$

$a + 2d = 16$ (1)

$a_7 - a_5 = 12$

$[a + (7 - 1) d] - [a + (5 - 1) d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obtain

$a + 2 (6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be

4, 10, 16, 22, …

Q17 :

Find the 20th term from the last term of the A.P. 3, 8, 13, …, 253

Answer :

Given A.P. is

3, 8, 13, …, 253

Common difference for this A.P. is 5.

Therefore, this A.P. can be written in reverse order as

253, 248, 243, …, 13, 8, 5

For this A.P.,

$a = 253$

$d = 248 - 253 = -5$

$n = 20$

$a_{20} = a + (20 - 1) d$

$a_{20} = 253 + (19) (-5)$

$a_{20} = 253 - 95$

$a = 158$

Therefore, 20th term from the last term is 158.

Q18 :

The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Answer :

We know that,

$a_n = a + (n - 1) d$

$a_4 = a + (4 - 1) d$

$a_4 = a + 3d$

Similarly,

$a_8 = a + 7d$

$a_6 = a + 5d$

$a_{10} = a + 9d$

Given that, $a_4 + a_8 = 24$

$a + 3d + a + 7d = 24$

$2a + 10d = 24$

$a + 5d = 12$ (1)

$a_6 + a_{10} = 44$

$a + 5d + a + 9d = 44$

$2a + 14d = 44$

$a + 7d = 22$ (2)

On subtracting equation (1) from (2), we obtain

$2d = 22 - 12$

$2d = 10$

$d = 5$

From equation (1), we obtain

$a + 5d = 12$

$a + 5 (5) = 12$

$a + 25 = 12$

$a = -13$

$a_2 = a + d = -13 + 5 = -8$

$a_3 = a_2 + d = -8 + 5 = -3$

Therefore, the first three terms of this A.P. are -13, -8, and -3.

Q19 :

Ramkali saved Rs 5 in the first week of a year and then increased her weekly saving by Rs 1.75. If in the n^{th}week, her week, her weekly savings become Rs 20.75, find *n*.

Answer :

Given that,

$a = 5$

$d = 1.75$

$a_n = 20.75$

$n = ?$

$a_n = a + (n - 1)\, d$

$$20.75 = 5 + (n-1)1.75$$

$$15.75 = (n-1)1.75$$

$$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$$= \frac{63}{7} = 9$$

$n - 1 = 9$

$n = 10$

Hence, *n* is 10.

Exercise 5.3 : Solutions of Questions on Page Number : **112**

Q1 :

Find the sum of the following APs.

(i) 2, 7, 12 ,…., to 10 terms.

(ii) - 37, - 33, - 29 ,…, to 12 terms

(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$ **,………, to 11 terms**

Answer :

(i)2, 7, 12 ,…, to 10 terms

For this A.P.,

$a = 2$

$d = a_2 - a_1 = 7 - 2 = 5$

$n = 10$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{10} = \frac{10}{2}\left[2(2) + (10-1)5\right]$$
$$= 5\left[4 + (9)\times(5)\right]$$
$$= 5\times 49 = 245$$

(ii) - 37, - 33, - 29 ,…, to 12 terms

For this A.P.,

$a = - 37$

$d = a_2 - a_1 = (- 33) - (- 37)$

$= - 33 + 37 = 4$

$n = 12$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{12} = \frac{12}{2}\left[2(-37) + (12-1)4\right]$$
$$= 6\left[-74 + 11\times 4\right]$$
$$= 6\left[-74 + 44\right]$$
$$= 6(-30) = -180$$

(iii) 0.6, 1.7, 2.8 ,…, to 100 terms

For this A.P.,

$a = 0.6$

$d = a_2 - a_1 = 1.7 - 0.6 = 1.1$

$n = 100$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(0.6) + (100-1)1.1\right]$$

$$= 50\left[1.2 + (99) \times (1.1)\right]$$

$$= 50\left[1.2 + 108.9\right]$$

$$= 50\left[110.1\right]$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10},$. …….. , to 11 terms

For this A.P.,

$$a = \frac{1}{15}$$

$n = 11$

$$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15}$$

$$= \frac{5-4}{60} = \frac{1}{60}$$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$$

Q2 :

Find the sums given below

(i) 7 + $10\frac{1}{2}$ + 14 + ………… + 84

(ii) 34 + 32 + 30 + ……….. + 10

(iii) - 5 + (- 8) + (- 11) + ………… + (- 230)

Answer :

(i)7 + $10\frac{1}{2}$ + 14 + …………+ 84

For this A.P.,

a = 7

l = 84

$$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$$

Let 84 be the n^{th} term of this A.P.

l = *a* + (*n* - 1)*d*

$$84 = 7 + (n-1)\frac{7}{2}$$

$$77 = (n-1)\frac{7}{2}$$

22 = *n* - 1

n = 23

We know that,

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{23}{2}[7+84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii)34 + 32 + 30 + ……….. + 10

For this A.P.,

$a = 34$

$d = a_2 - a_1 = 32 - 34 = -2$

$l = 10$

Let 10 be the n^{th} term of this A.P.

$l = a + (n - 1)d$

$10 = 34 + (n - 1)(-2)$

$-24 = (n - 1)(-2)$

$12 = n - 1$

$n = 13$

$$S_n = \frac{n}{2}(a + l)$$

$$= \frac{13}{2}(34 + 10)$$

$$= \frac{13 \times 44}{2} = 13 \times 22$$

$$= 286$$

(iii)$(-5) + (-8) + (-11) + \ldots\ldots\ldots\ldots + (-230)$

For this A.P.,

$a = -5$

$l = -230$

$d = a_2 - a_1 = (-8) - (-5)$

$= -8 + 5 = -3$

Let -230 be the n^{th} term of this A.P.

$l = a + (n - 1)d$

$-230 = -5 + (n - 1)(-3)$

$-225 = (n - 1)(-3)$

$(n - 1) = 75$

$n = 76$

And, $S_n = \frac{n}{2}(a + l)$

$$= \frac{76}{2}\left[(-5)+(-230)\right]$$
$$= 38(-235)$$
$$= -8930$$

Q3 :

In an AP

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = - 14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

(x)Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Answer :

(i) Given that, $a = 5$, $d = 3$, $a_n = 50$

As $a_n = a + (n - 1)d$,

$\therefore 50 = 5 + (n - 1)3$

$45 = (n - 1)3$

$15 = n - 1$

$n = 16$

$$S_n = \frac{n}{2}\left[a + a_n\right]$$
$$S_{16} = \frac{16}{2}\left[5 + 50\right]$$
$$= 8 \times 55$$
$$= 440$$

(ii) Given that, $a = 7$, $a_{13} = 35$

As $a_n = a + (n - 1) d$,

$\therefore a_{13} = a + (13 - 1) d$

$35 = 7 + 12 d$

$35 - 7 = 12d$

$28 = 12d$

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$\begin{aligned} S_{13} &= \frac{n}{2}[a + a_{13}] \\ &= \frac{13}{2}[7 + 35] \\ &= \frac{13 \times 42}{2} = 13 \times 21 \\ &= 273 \end{aligned}$$

(iii)Given that, $a_{12} = 37$, $d = 3$

As $a_n = a + (n - 1)d$,

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_n = \frac{12}{2}[4 + 37]$$

$$S_n = 6(41)$$

$$S_n = 246$$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As $a_n = a + (n - 1)d$,

$a_3 = a + (3 - 1)d$

$15 = a + 2d$ (i)

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2a + (10-1)d\right]$$

$$125 = 5(2a + 9d)$$

$$25 = 2a + 9d \qquad \text{(ii)}$$

On multiplying equation (1) by 2, we obtain

$30 = 2a + 4d$ (iii)

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$

$a_{10} = 17 + (9)(-1)$

$a_{10} = 17 - 9 = 8$

(v)Given that, $d = 5$, $S_9 = 75$

As $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$$S_9 = \frac{9}{2}\left[2a + (9-1)5\right]$$

$$75 = \frac{9}{2}(2a + 40)$$

$25 = 3(a + 20)$

$25 = 3a + 60$

$3a = 25 - 60$

$$a = \frac{-35}{3}$$

$a_n = a + (n - 1)d$

$a_9 = a + (9 - 1)(5)$

$$= \frac{-35}{3} + 8(5)$$

$$= \frac{-35}{3} + 40$$

$$= \frac{-35+120}{3} = \frac{85}{3}$$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

As $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$,

$$90 = \frac{n}{2}\left[4 + (n-1)8\right]$$

$90 = n [2 + (n - 1)4]$

$90 = n [2 + 4n - 4]$

$90 = n (4n - 2) = 4n^2 - 2n$

$4n^2 - 2n - 90 = 0$

$4n^2 - 20n + 18n - 90 = 0$

$4n (n - 5) + 18 (n - 5) = 0$

$(n - 5) (4n + 18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

a

Q4 :

How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Answer :

Let there be n terms of this A.P.

For this A.P., $a = 9$

$d = a_2 - a_1 = 17 - 9 = 8$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$636 = \frac{n}{2}\left[2 \times a + (n-1)8\right]$$

$$636 = \frac{n}{2}\left[18 + (n-1)8\right]$$

$636 = n\,[9 + 4n - 4]$

$636 = n\,(4n + 5)$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$n\,(4n + 53) - 12\,(4n + 53) = 0$

$(4n + 53)\,(n - 12) = 0$

Either $4n + 53 = 0$ or $n - 12 = 0$

$n = \frac{-53}{4}$ or $n = 12$

n cannot be $-\frac{53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$ only.

Q5 :

The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer :

Given that,

$a = 5$

$l = 45$

$S_n = 400$

$$S_n = \frac{n}{2}(a + l)$$

$$400 = \frac{n}{2}(5 + 45)$$

$$400 = \frac{n}{2}(50)$$

$n = 16$

$l = a + (n - 1)\, d$

$45 = 5 + (16 - 1)\, d$

$40 = 15d$

$$d = \frac{40}{15} = \frac{8}{3}$$

Q6 :

The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer :

Given that,

$a = 17$

$l = 350$

$d = 9$

Let there be n terms in the A.P.

$l = a + (n - 1)\, d$

$350 = 17 + (n - 1)9$

$333 = (n - 1)9$

$(n - 1) = 37$

$n = 38$

$$S_n = \frac{n}{2}(a+l)$$

$$\Rightarrow S_n = \frac{38}{2}(17+350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Q7 :

Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Answer :

$d = 7$

$a_{22} = 149$

$S_{22} = ?$

$a_n = a + (n - 1)d$

$a_{22} = a + (22 - 1)d$

$149 = a + 21 \times 7$

$149 = a + 147$

$a = 2$

$$\begin{aligned} S_n &= \frac{n}{2}\left(a + a_n\right) \\ &= \frac{22}{2}\left(2 + 149\right) \\ &= 11\left(151\right) = 1661 \end{aligned}$$

Q8 :

Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer :

Given that,

$a_2 = 14$

$a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d$

$14 = a + 4$

$a = 10$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{51} = \frac{51}{2}\left[2\times10+(51-1)4\right]$$

$$= \frac{51}{2}\left[20+(50)(4)\right]$$

$$= \frac{51(220)}{2} = 51(110)$$

= 5610

Q9 :

If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first *n* terms.

Answer :

Given that,

$S_7 = 49$

$S_{17} = 289$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_7 = \frac{7}{2}\left[2a+(7-1)d\right]$$

$$49 = \frac{7}{2}(2a+6d)$$

$7 = (a + 3d)$

$a + 3d = 7$ (*i*)

Similarly, $S_{17} = \frac{17}{2}\left[2a+(17-1)d\right]$

$$289 = \frac{17}{2}\left[2a+16d\right]$$

$17 = (a + 8d)$

$a + 8d = 17$ (*ii*)

Subtracting equation (*i*) from equation (*ii*),

$5d = 10$

$d = 2$

From equation (i),

$a + 3(2) = 7$

$a + 6 = 7$

$a = 1$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2(1) + (n-1)(2)\right]$$

$$= \frac{n}{2}(2 + 2n - 2)$$

$$= \frac{n}{2}(2n)$$

$= n^2$

Q10 :

Show that a_1, a_2 … , a_n , … form an AP where a_n is defined as below

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Answer :

(i) $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3 + 12 = 15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

It can be observed that

$a_2 - a_1 = 11 - 7 = 4$

$a_3 - a_2 = 15 - 11 = 4$

$a_4 - a_3 = 19 - 15 = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(7)+(15-1)4\right]$$

$$= \frac{15}{2}\left[(14)+56\right]$$

$$= \frac{15}{2}(70)$$

= 15 × 35

= 525

(ii) $a_n = 9 - 5n$

$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$

$a_2 = 9 - 5 \times 2 = 9 - 10 = - 1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = - 6$

$a_4 = 9 - 5 \times 4 = 9 - 20 = - 11$

It can be observed that

$a_2 - a_1 = - 1 - 4 = - 5$

$a_3 - a_2 = - 6 - (- 1) = - 5$

$a_4 - a_3 = - 11 - (- 6) = - 5$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as - 5 and first term as 4.

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(4)+(15-1)(-5)\right]$$

$$= \frac{15}{2}\left[8+14(-5)\right]$$

$$= \frac{15}{2}(8-70)$$

$$= \frac{15}{2}(-62) = 15(-31)$$

= - 465

Q11 :

If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3rd, the10th and the n^{th} terms.

Answer :

Given that,

$S_n = 4n - n^2$

First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms = S_2

$= 4(2) - (2)^2 = 8 - 4 = 4$

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$

$= 3 + (n - 1) (-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1. 3rd, 10th, and n^{th} terms are -1, -15, and $5 - 2n$ respectively.

Q12 :

Find the sum of first 40 positive integers divisible by 6.

Answer :

The positive integers that are divisible by 6 are

6, 12, 18, 24 …

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

$a = 6$

$d = 6$

$S_{40} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(6) + (40-1)6\right]$$

= 20[12 + (39) (6)]

= 20(12 + 234)

= 20 × 246

= 4920

Q13 :

Find the sum of first 15 multiples of 8.

Answer :

The multiples of 8 are

8, 16, 24, 32…

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, *a* = 8

d = 8

S_{15} =?

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{15}{2}\left[2(8) + (15-1)8\right]$$

$$= \frac{15}{2}\left[16 + 14(8)\right]$$

$$= \frac{15}{2}(16 + 112)$$

$$= \frac{15(128)}{2} = 15 \times 64$$

= 960

Q14 :

Find the sum of the odd numbers between 0 and 50.

Answer :

The odd numbers between 0 and 50 are

1, 3, 5, 7, 9 … 49

Therefore, it can be observed that these odd numbers are in an A.P.

$a = 1$

$d = 2$

$l = 49$

$l = a + (n - 1)\ d$

$49 = 1 + (n - 1)2$

$48 = 2(n - 1)$

$n - 1 = 24$

$n = 25$

$$S_n = \frac{n}{2}(a+l)$$

$$S_{25} = \frac{25}{2}(1+49)$$
$$= \frac{25(50)}{2} = (25)(25)$$

= 625

Q15 :

A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Answer :

It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$a = 200$

$d = 50$

Penalty that has to be paid if he has delayed the work by 30 days = S_{30}

$$= \frac{30}{2}\left[2(200)+(30-1)50\right]$$

= 15 [400 + 1450]

= 15 (1850)

= 27750

Therefore, the contractor has to pay Rs 27750 as penalty.

Q16 :

A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer :

Let the cost of 1st prize be P.

Cost of 2nd prize = P - 20

And cost of 3rd prize = P - 40

It can be observed that the cost of these prizes are in an A.P. having common difference as - 20 and first term as P.

$a = P$

$d = - 20$

Given that, $S_7 = 700$

$$\frac{7}{2}\left[2a+(7-1)d\right] = 700$$

$$\frac{\left[2a+(6)(-20)\right]}{2} = 100$$

$a + 3(- 20) = 100$

$a - 60 = 100$

$a = 160$

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Q17 :

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in

which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Answer :

It can be observed that the number of trees planted by the students is in an AP.

1, 2, 3, 4, 5………………..12

First term, *a* = 1

Common difference, *d* = 2 - 1 = 1

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(1) + (12-1)(1)\right]$$

= 6 (2 + 11)

= 6 (13)

= 78

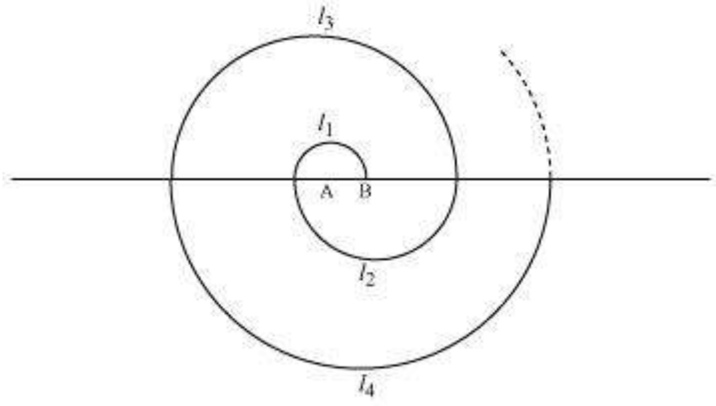
Therefore, number of trees planted by 1 section of the classes = 78

Number of trees planted by 3 sections of the classes = 3 × 78 = 234

Therefore, 234 trees will be planted by the students.

Q18 :

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? $\left(\text{Take } \pi = \frac{22}{7}\right)$



Answer :

Semi-perimeter of circle = π*r*

$I_1 = \pi(0.5) = \frac{\pi}{2}$ cm

$I_2 = \pi(1) = \pi$ cm

$I_3 = \pi(1.5) = \frac{3\pi}{2}$ cm

Therefore, I_1, I_2, I_3 ,i.e. the lengths of the semi-circles are in an A.P.,

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots\ldots\ldots\ldots$$

$$a = \frac{\pi}{2}$$

$$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

S_{13} =?

We know that the sum of *n* terms of an a A.P. is given by

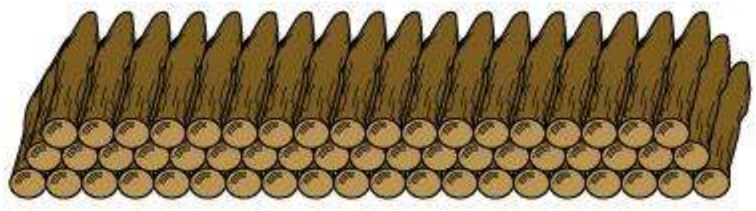
$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ &= \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right] \\ &= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right] \\ &= \left(\frac{13}{2}\right)(7\pi) \\ &= \frac{91\pi}{2} \\ &= \frac{91 \times 22}{2 \times 7} = 13 \times 11 \end{aligned}$$

= 143

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Q19 :

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Answer :

It can be observed that the numbers of logs in rows are in an A.P.

20, 19, 18…

For this A.P.,

$a = 20$

$d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$200 = \frac{n}{2}\left[2(20) + (n-1)(-1)\right]$$

$400 = n(40 - n + 1)$

$400 = n(41 - n)$

$400 = 41n - n^2$

$n^2 - 41n + 400 = 0$

$n^2 - 16n - 25n + 400 = 0$

$n(n - 16) - 25(n - 16) = 0$

$(n - 16)(n - 25) = 0$

Either $(n - 16) = 0$ or $n - 25 = 0$

$n = 16$ or $n = 25$

$a_n = a + (n - 1)d$

$a_{16} = 20 + (16 - 1)(-1)$

$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly,

$a_{25} = 20 + (25 - 1)(-1)$

$a_{25} = 20 - 24$

$= -4$

Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

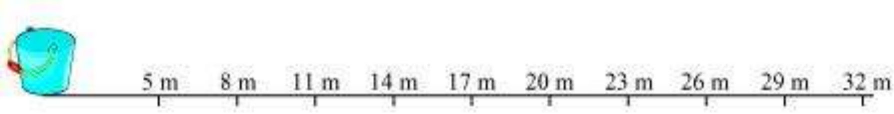
Q20 :

In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is 2 x 5 + 2 x (5 + 3)]

Answer :



The distances of potatoes are as follows.

5, 8, 11, 14…

It can be observed that these distances are in A.P.

$a = 5$

$d = 8 - 5 = 3$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(5) + (10-1)3\right]$$

= 5[10 + 9 × 3]

= 5(10 + 27) = 5(37)

= 185

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

Therefore, total distance that the competitor will run = 2 × 185

= 370 m

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14…

Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept. Therefore, distances to be run are

10, 16, 22, 28, 34,……….

$a = 10$

$d = 16 - 10 = 6$

$S_{10} = ?$

$$S_{10} = \frac{10}{2}\left[2\times 10+\left(10-1\right)6\right]$$

= 5[20 + 54]

= 5 (74)

= 370

Therefore, the competitor will run a total distance of 370 m.

Exercise 5.4 : Solutions of Questions on Page Number : **115**

Q1 :

Which term of the A.P. 121, 117, 113 … is its first negative term?

[Hint: Find n for $a_n < 0$]

Answer :

Given A.P. is 121, 117, 113 …

$a = 121$

$d = 117 - 121 = - 4$

$a_n = a + (n - 1) d$

$= 121 + (n - 1)(-4)$

$= 121 - 4n + 4$

$= 125 - 4n$

We have to find the first negative term of this A.P.

$$\text{Therefore}, a_n < 0$$
$$125 - 4n < 0$$
$$125 < 4n$$
$$n > \frac{125}{4}$$
$$n > 31.25$$

Therefore, 32nd term will be the first negative term of this A.P.

Q2 :

The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Answer :

We know that,

$a_n = a + (n - 1)d$

$a_3 = a + (3 - 1)d$

$a_3 = a + 2d$

Similarly, $a_7 = a + 6d$

Given that, $a_3 + a_7 = 6$

$(a + 2d) + (a + 6d) = 6$

$2a + 8d = 6$

$a + 4d = 3$

$a = 3 - 4d \quad (i)$

Also, it is given that $(a_3) \times (a_7) = 8$

$(a + 2d) \times (a + 6d) = 8$

From equation (i),

$$(3-4d+2d)\times(3-4d+6d)=8$$
$$(3-2d)\times(3+2d)=8$$
$$9-4d^2=8$$
$$4d^2=9-8=1$$
$$d^2=\frac{1}{4}$$
$$d=\pm\frac{1}{2}$$
$$d=\frac{1}{2}\text{ or }-\frac{1}{2}$$

From equation (*i*),

$$\left(\text{When } d \text{ is } \frac{1}{2}\right)$$
$$a=3-4d$$
$$a=3-4\left(\frac{1}{2}\right)$$
$$=3-2=1$$
$$\left(\text{When } d \text{ is } -\frac{1}{2}\right)$$
$$a=3-4\left(-\frac{1}{2}\right)$$
$$a=3+2=5$$
$$S_n=\frac{n}{2}\left[2a(n-1)d\right]$$
$$\left(\text{When } a \text{ is} 1 \text{ and } d \text{ is } \frac{1}{2}\right)$$
$$S_{16}=\frac{16}{2}\left[2(1)+(16-1)\left(\frac{1}{2}\right)\right]$$
$$=8\left[2+\frac{15}{2}\right]$$
$$=4(19)=76$$
$$\left(\text{When } a \text{ is 5 and } d \text{ is } -\frac{1}{2}\right)$$

$$S_{16} = \frac{16}{2}\left[2(5)+(16-1)\left(-\frac{1}{2}\right)\right]$$

$$= 8\left[10+(15)\left(-\frac{1}{2}\right)\right]$$

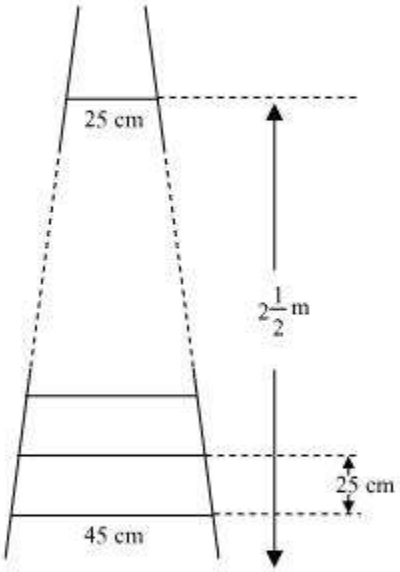
$$= 8\left(\frac{5}{2}\right)$$

$$= 20$$

Q3 :

A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[Hint: number of rungs $= \frac{250}{25}$]



Answer :

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

∴ Total number of rungs $= \dfrac{2\frac{1}{2}\times 100}{25} + 1 = \dfrac{250}{25} + 1 = 11$

Now, as the lengths of the rungs decrease uniformly, they will be in an A.P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

First term, $a = 45$

Last term, $l = 25$

$n = 11$

$$S_n = \frac{n}{2}(a+l)$$

$$\therefore S_{10} = \frac{11}{2}(45+25) = \frac{11}{2}(70) = 385 \text{ cm}$$

Therefore, the length of the wood required for the rungs is 385 cm.

Q4 :

The houses of a row are number consecutively from 1 to 49. Show that there is a value of *x* such that the sum of numbers of the houses preceding the house numbered *x* is equal to the sum of the number of houses following it.

Find this value of *x*.

[Hint $S_{x-1} = S_{49} - S_x$]

Answer :

The number of houses was

1, 2, 3 … 49

It can be observed that the number of houses are in an A.P. having *a* as 1 and *d* also as 1.

Let us assume that the number of x^{th} house was like this.

We know that,

Sum of *n* terms in an A.P. $= \frac{n}{2}\left[2a+(n-1)d\right]$

Sum of number of houses preceding x^{th} house $= S_{x-1}$

$$= \frac{(x-1)}{2}\left[2a+(x-1-1)d\right]$$

$$= \frac{x-1}{2}\left[2(1)+(x-2)(1)\right]$$

$$= \frac{x-1}{2}\left[2+x-2\right]$$

$$= \frac{(x)(x-1)}{2}$$

Sum of number of houses following x^{th} house = S_{49} - S_x

$$= \frac{49}{2}\left[2(1)+(49-1)(1)\right]-\frac{x}{2}\left[2(1)+(x-1)(1)\right]$$

$$= \frac{49}{2}(2+49-1)-\frac{x}{2}(2+x-1)$$

$$= \left(\frac{49}{2}\right)(50)-\frac{x}{2}(x+1)$$

$$= 25(49)-\frac{x(x+1)}{2}$$

It is given that these sums are equal to each other.

$$\frac{x(x-1)}{2} = 25(49)-x\left(\frac{x+1}{2}\right)$$

$$\frac{x^2}{2}-\frac{x}{2} = 1225-\frac{x^2}{2}-\frac{x}{2}$$

$$x^2 = 1225$$

$$x = \pm 35$$

However, the house numbers are positive integers.

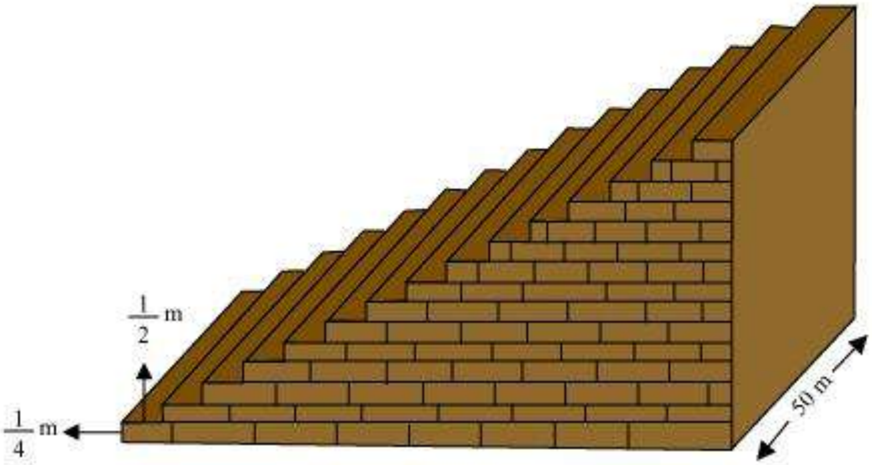
The value of x will be 35 only.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

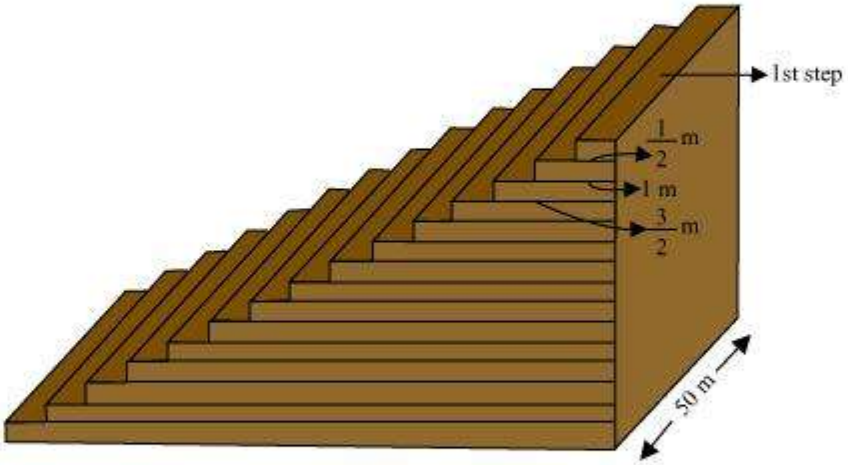
Q5 :

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (See figure) calculate the total volume of concrete required to build the terrace.



Answer :



From the figure, it can be observed that

1st step is $\frac{1}{2}$ m wide,

2nd step is 1 m wide,

3rd step is $\frac{3}{2}$ m wide.

Therefore, the width of each step is increasing by $\frac{1}{2}$ m each time whereas their height $\frac{1}{4}$ m and length 50 m remains the same.

Therefore, the widths of these steps are

$$\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

Volume of concrete in 1st step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete in 2nd step $= \frac{1}{4} \times 1 \times 50 = \frac{25}{2}$

Volume of concrete in 3rd step $= \frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in an A.P.

$$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$$

$$a = \frac{25}{4}$$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

$$\text{and } S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\frac{25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{(14)25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$$

$$= \frac{15}{2}(100) = 750$$

Volume of concrete required to build the terrace is 750 m^3.

DELHI PUBLIC SCHOOL, GANDHINAGAR

CHAPTER 6: TRIANGLES

MIND MAP

This chapter consists of seven different topics. The most probable questions from examination point of view are given below.

TYPE: 1 SIMILARITY OF TRIANGLES

Q.1. If $\Delta ABC \sim \Delta PQR$, AB= $5cm, BC = 6cm, PQ = 8cm$, then find QR.

TYPE: 2 BASIC PROPORTIONALITY THEOREM

Q.1. State and prove BPT.

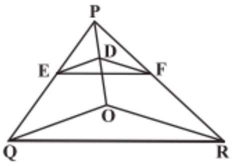
Q.2. In $\Delta ABC, D$ and E are points on the sides AB and AC respectively, such that $DE \parallel BC$. If $AD = x, DB = x - 2, AE = x + 2$ and $EC = x - 1$, find the value of x.

Q.3. In $\Delta ABC, DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$

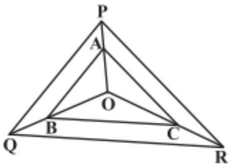
TYPE: 3 CONVERSE OF BASIC PROPORTIONALITY THEOREM

Q.1. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR for PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

Q.2. In the adjoining figure, DE || OQ and DF || OR. Show that EF || QR.



Q.3. In the adjoining figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

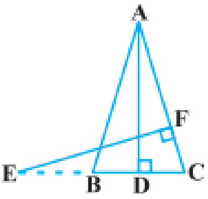


TYPE: 4 CRITERIA FOR SIMILARITY OF TRIANGLES

Q.1. S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$.

Show that $\Delta RPQ \sim \Delta RTS$.

Q.2. In the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If $AD \perp BC$ and $EF \perp AC$, prove that $\Delta ABD \sim \Delta ECF$.



Q.3. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.

TYPE: 5 AREAS OF SIMILAR TRIANGLES

Q.1. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Q.2. If $\Delta ABC \sim \Delta DEF, BC = 3cm\ EF = 4cm$ and area of $\Delta ABC = 54cm^2$. Determine the area of ΔDEF.

Q.3. If the areas of two similar triangles are in ratio $25 : 64$, write the ratio of their corresponding sides.

Q.4. Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

TYPE: 6 PYTHAGORAS THEOREM

Q.1. State and prove Pythagoras theorem.

Q.2. In an equilateral $\Delta ABC, D$ is a point on the side BC such that $BD = \frac{1}{3}BC$.

Prove that $9AD^2 = 7AB^2$.

Q.3. BL and CM are medians of a triangle ABC right angled at A.

Prove that $4(BL^2 + CM^2) = 5BC^2$.

TYPE: 7 CONVERSE OF PYTHAGORAS THEOREM

Q.1. State and prove converse of Pythagoras theorem.

Q.2. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Chapter 6: Triangles

Unit 6 Triangles Exercise 6.1, 6.2, 6.3, 6.4, 6.5, 6.6 Solutions

Exercise 6.1 : Solutions of Questions on Page Number : **122**

Q1 :

Fill in the blanks using correct word given in the brackets:-

(i) All circles are __________. (congruent, similar)

(ii) All squares are __________. (similar, congruent)

(iii) All __________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are __________ and (b) their corresponding sides are __________. (equal, proportional)

Answer :

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

Q2 :

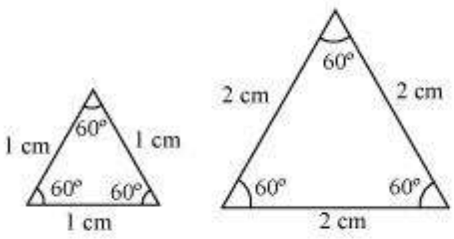
Give two different examples of pair of

(i) Similar figures

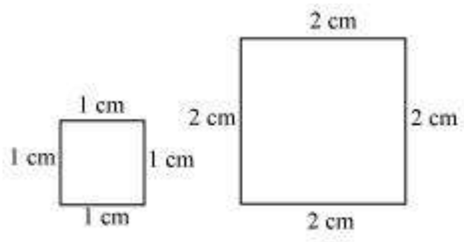
(ii)Non-similar figures

Answer :

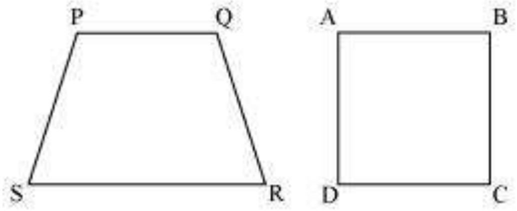
(i) Two equilateral triangles with sides 1 cm and 2 cm



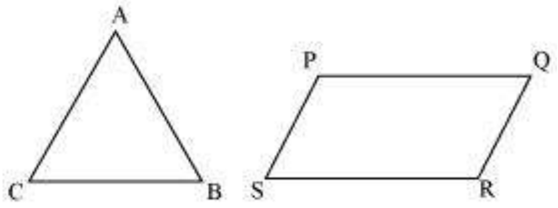
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

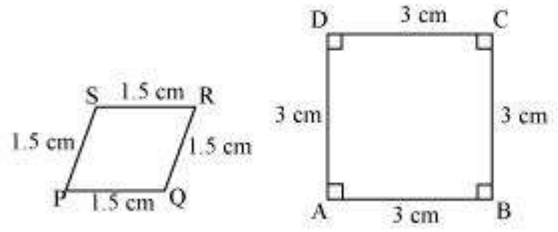


Triangle and parallelogram



Q3 :

State whether the following quadrilaterals are similar or not:



Answer :

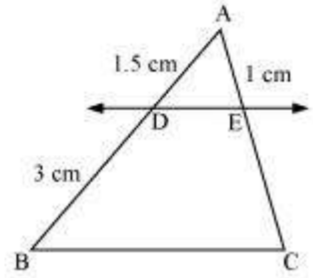
Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Exercise 6.2 : Solutions of Questions on Page Number : **128**

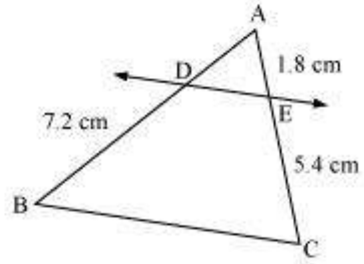
Q1 :

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

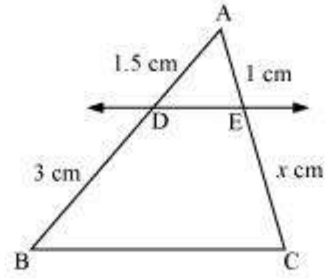


(ii)



Answer :

(i)



Let EC = *x* cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{\text{AD}}{\text{DB}}=\frac{\text{AE}}{\text{EC}}$$

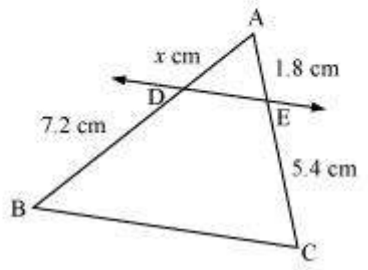
$$\frac{1.5}{3}=\frac{1}{x}$$

$$x=\frac{3\times 1}{1.5}$$

$$x=2$$

$$\therefore \text{EC}=2\ \text{cm}$$

(ii)



Let AD = *x* cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4 \text{ cm}$$

Q2 :

E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

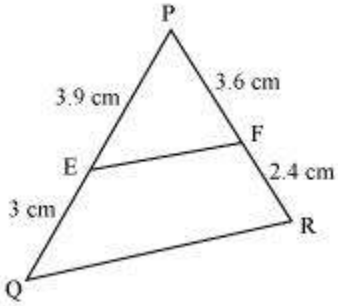
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer :

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

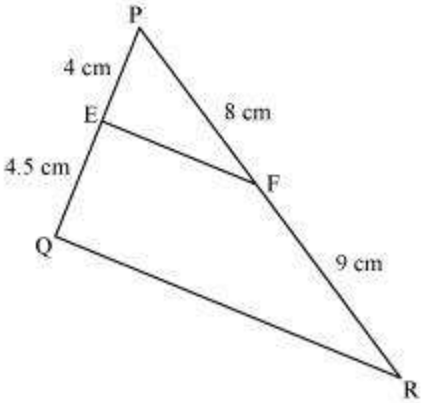
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

Hence, $\frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

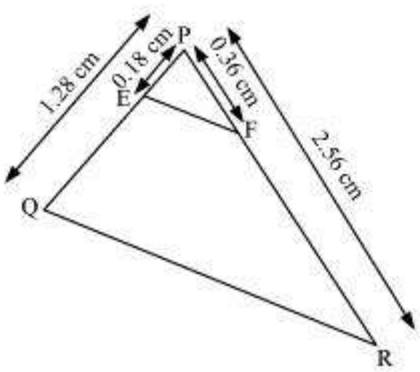
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

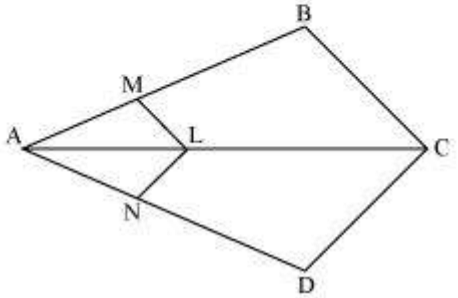
Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

Therefore, EF is parallel to QR.

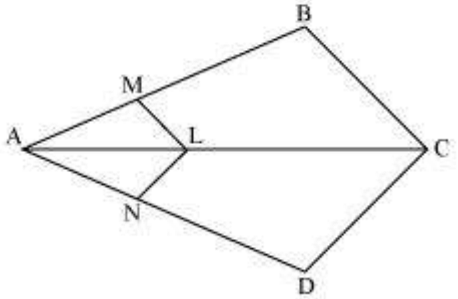
Q3 :

In the following figure, if LM || CB and LN || CD, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}.$$



Answer :



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \qquad (i)$$

Similarly, LN || CD

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \qquad (ii)$$

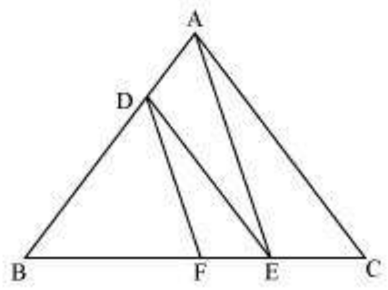
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

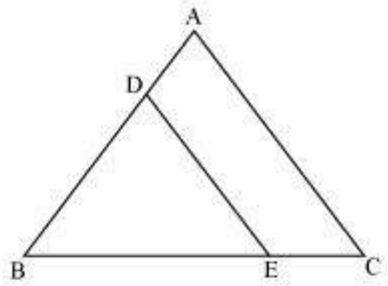
Q4 :

In the following figure, DE || AC and DF || AE. Prove that

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

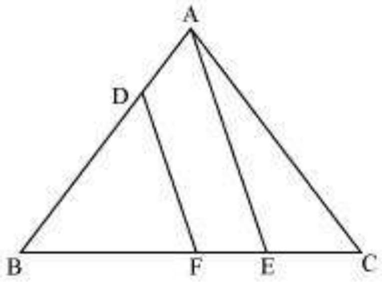


Answer :



In ΔABC, DE || AC

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \qquad \text{(Basic Proportionality Theorem)} \qquad (i)$$



In ΔBAE, $DF \parallel AE$

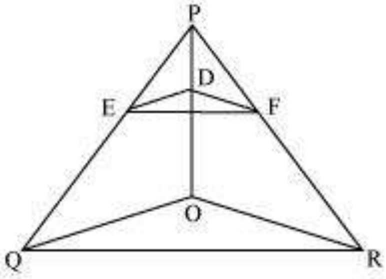
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \qquad \text{(Basic Proportionality Theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

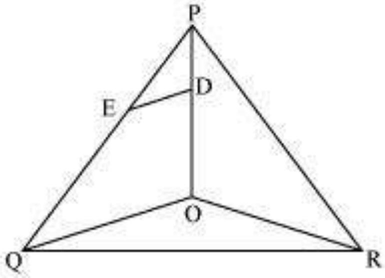
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Q5 :

In the following figure, DE || OQ and DF || OR, show that EF || QR.

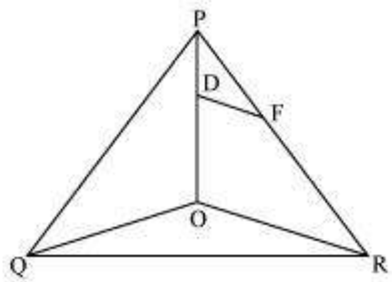


Answer :



In Δ POQ, DE || OQ

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



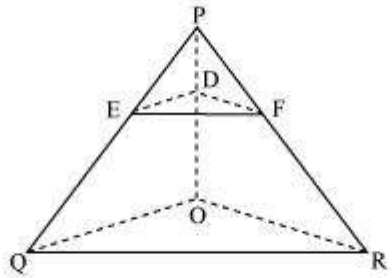
In ΔPOR, $DF \parallel OR$

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we obtain

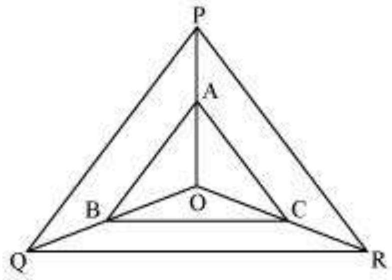
$\frac{PE}{EQ} = \frac{PF}{FR}$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

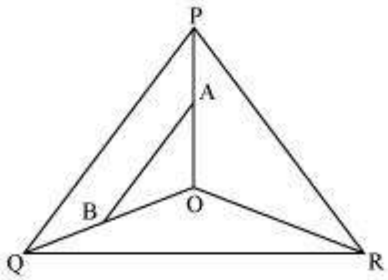


Q6 :

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

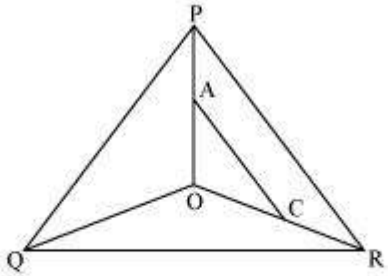


Answer :



In Δ POQ, AB || PQ

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



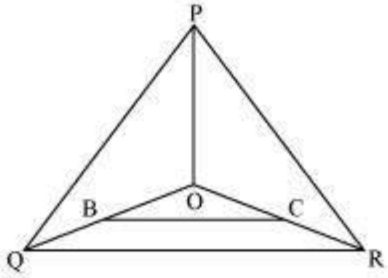
In ΔPOR, AC || PR

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \qquad \text{(By basic proportionality theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

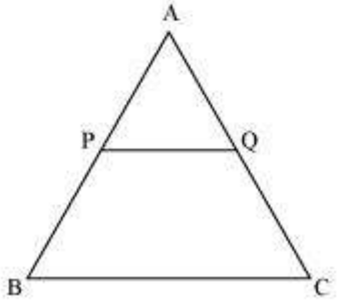
$\therefore$ BC || QR (By the converse of basic proportionality theorem)



Q7 :

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad (\text{P is the mid-point of AB. } \therefore \text{ AP} = \text{PB})$$

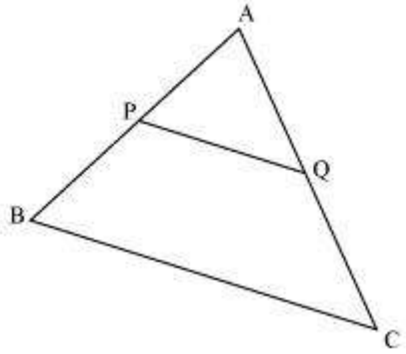
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Q8 :

Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer :



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC

It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

$$\text{and } \frac{AQ}{QC} = \frac{1}{1}$$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

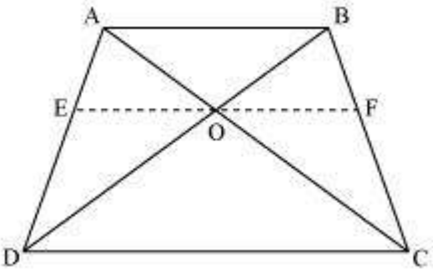
Hence, by using basic proportionality theorem, we obtain

$$PQ \| BC$$

Q9 :

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer :



Draw a line EF through point O, such that $EF \| CD$

In ΔADC, $EO \| CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

In ΔABD, $OE \| AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

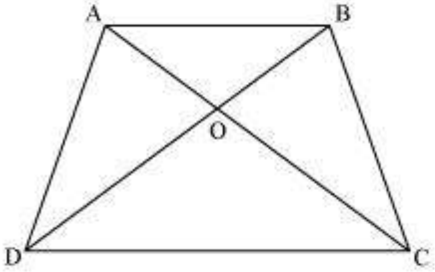
$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

Q10 :

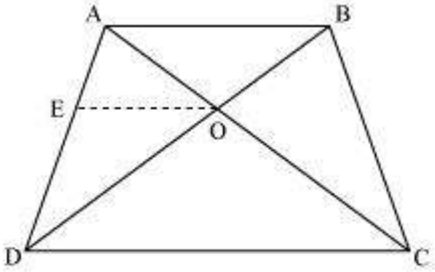
The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer :

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

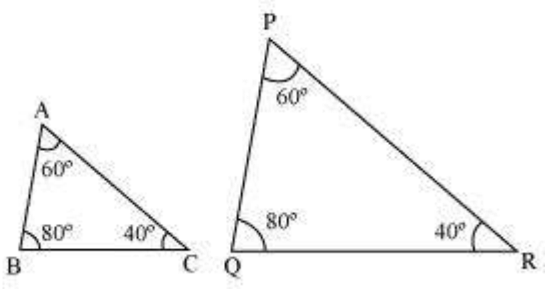
∴ ABCD is a trapezium.

Exercise 6.3 : Solutions of Questions on Page Number : **138**

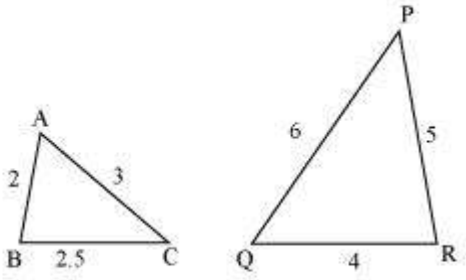
Q1 :

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

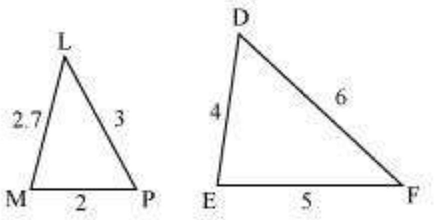
(i)



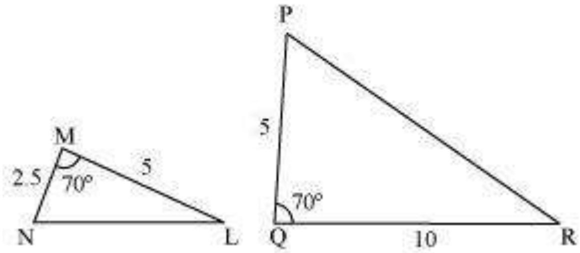
(ii)



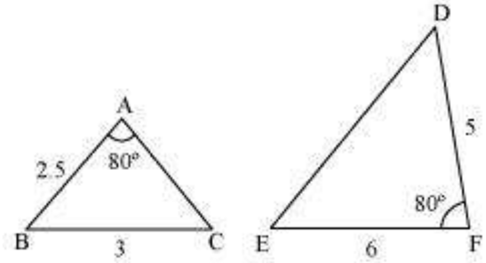
(iii)



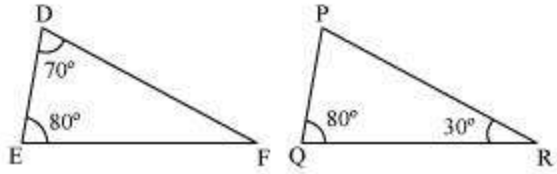
(iv)



(v)



(vi)



Answer :

(i) ∠A = ∠P = 60°

∠B = ∠Q = 80°

∠C = ∠R = 40°

Therefore, ΔABC Ã¢Ë†Â¼ ΔPQR [By AAA similarity criterion]

$$\frac{AB}{QR}=\frac{BC}{RP}=\frac{CA}{PQ}$$

(ii)

$$\therefore\ \Delta ABC \sim \Delta QRP \qquad \left[\text{By SSS similarity criterion}\right]$$

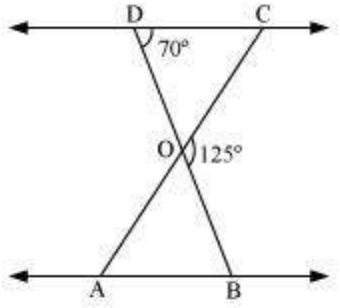
(iii)The given triangles are not similar as the corresponding sides are not proportional.

(iv) In Ã¢Ë†â€ MNL and Ã¢Ë†â€ QPR, we observe that,

MNQP = MLQR = 12

Q2 :

In the following figure, ΔODC ∝¼ ΔOBA, ∠ BOC = 125° and ∠ CDO = 70°. Find ∠ DOC, ∠ DCO and ∠ OAB



Answer :

DOB is a straight line.

∴ ∠ DOC + ∠ COB = 180°

⇒ ∠ DOC = 180° - 125°

= 55°

In ΔDOC,

∠ DCO + ∠ CDO + ∠ DOC = 180°

(Sum of the measures of the angles of a triangle is 180º.)

⇒ ∠ DCO + 70º + 55º = 180°

⇒ ∠ DCO = 55°

It is given that ΔODC ∠Â¼ ΔOBA.

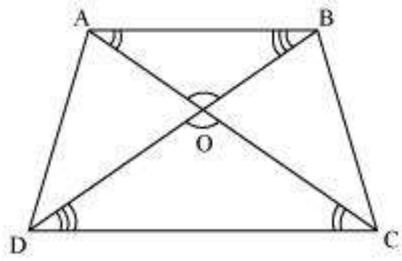
∴ ∠ OAB = ∠ OCD [Corresponding angles are equal in similar triangles.]

⇒ ∠ OAB = 55°

Q3 :

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer :



In ΔDOC and ΔBOA,

∠CDO = ∠ABO [Alternate interior angles as AB || CD]

∠DCO = ∠BAO [Alternate interior angles as AB || CD]

∠DOC = ∠BOA [Vertically opposite angles]

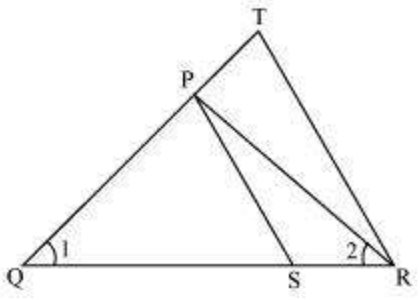
∴ ΔDOC Ã¢Ë†Â¼ ΔBOA [AAA similarity criterion]

$$\therefore \frac{DO}{BO} = \frac{OC}{OA} \qquad \left[\text{Corresponding sides are proportional}\right]$$

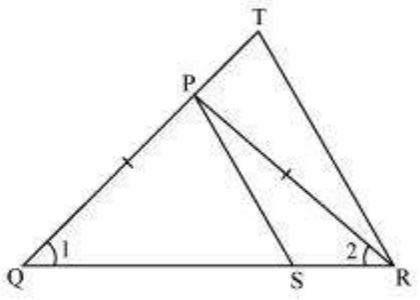
$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

Q4 :

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2.$ **Show that** $\Delta PQS \sim \Delta TQR$



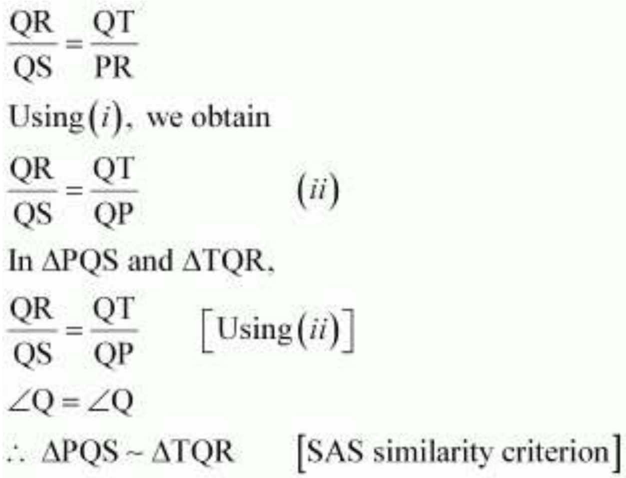
Answer :



In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR (i)

Given,



$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad (ii)$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad [\text{Using}(ii)]$$

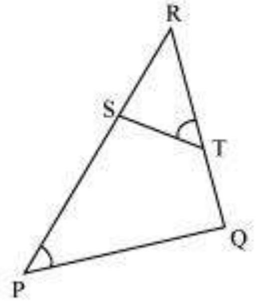
$$\angle Q = \angle Q$$

$$\therefore\ \Delta PQS \sim \Delta TQR \qquad [\text{SAS similarity criterion}]$$

Q5 :

S and T are point on sides PR and QR of ΔPQR such that ∠ P = ∠ RTS. Show that ΔRPQ ∠Â¼ ΔRTS.

Answer :



In ΔRPQ and ΔRST,

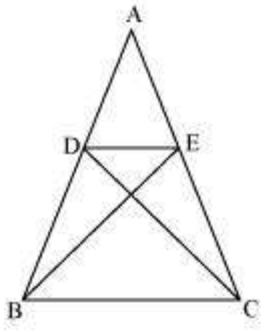
∠ RTS = ∠ QPS (Given)

∠ R = ∠ R (Common angle)

∴ ΔRPQ ∝¼ ΔRTS (By AA similarity criterion)

Q6 :

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ∝¼ ΔABC.



Answer :

It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] (1)

And, AD = AE [By CPCT] (2)

In ΔADE and ΔABC,

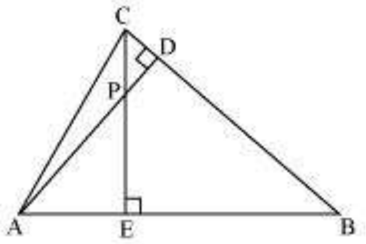
$$\frac{AD}{AB} = \frac{AE}{AC}$$ [Dividing equation (2) by (1)]

∠A = ∠A [Common angle]

∴ ΔADE Ã¢Ë†Â¼ ΔABC [By SAS similarity criterion]

Q7 :

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ∝¼ ΔCDP

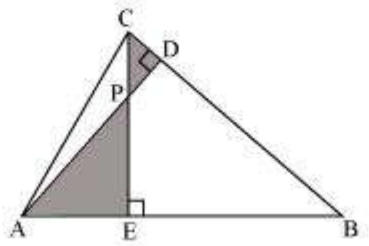
(ii) ΔABD ∝¼ ΔCBE

(iii) ΔAEP ∝¼ ΔADB

(v) ΔPDC ∝¼ ΔBEC

Answer :

(i)



In ΔAEP and ΔCDP,

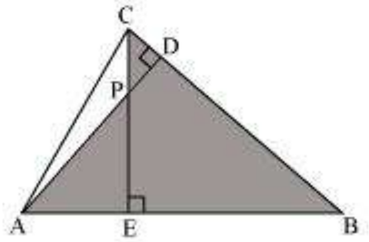
∠ AEP = ∠ CDP (Each 90°)

∠ APE = ∠ CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ∝¼ ΔCDP

(ii)



In ΔABD and ΔCBE,

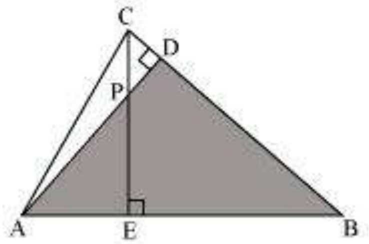
∠ ADB = ∠ CEB (Each 90°)

∠ ABD = ∠ CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ∝¼ ΔCBE

(iii)



In ΔAEP and ΔADB,

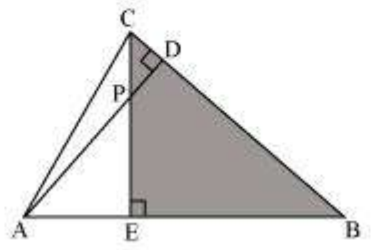
∠ AEP = ∠ ADB (Each 90°)

∠ PAE = ∠ DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ∝¼ ΔADB

(iv)



In ΔPDC and ΔBEC,

∠ PDC = ∠ BEC (Each 90°)

∠ PCD = ∠ BCE (Common angle)

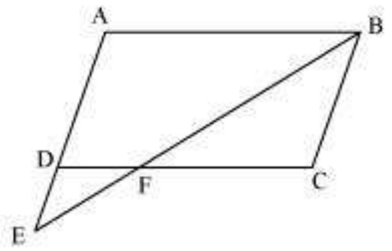
Hence, by using AA similarity criterion,

ΔPDC ∝¼ ΔBEC

Q8 :

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ∠Â¼ ΔCFB

Answer :



In ΔABE and ΔCFB,

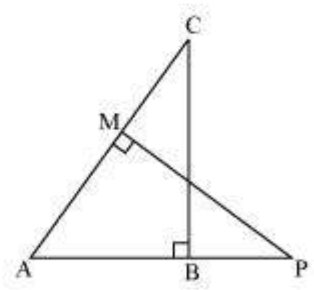
∠ A = ∠ C (Opposite angles of a parallelogram)

∠ AEB = ∠ CBF (Alternate interior angles as AE || BC)

∴ ΔABE ∝¼ ΔCFB (By AA similarity criterion)

Q9 :

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) ΔABC Ã¢Ë†Â¼ ΔAMP

(ii) $\frac{CA}{PA}=\frac{BC}{MP}$

Answer :

In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ ΔABC Ã¢Ë†Â¼ ΔAMP (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA}=\frac{BC}{MP} \qquad \text{(Corresponding sides of similar triangles are proportional)}$$

Q10 :

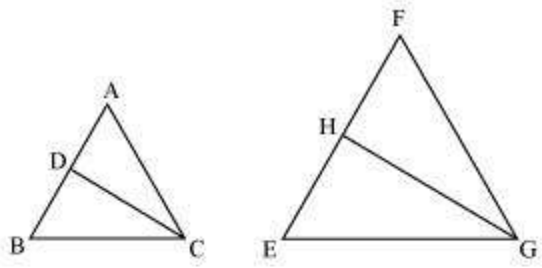
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC Ã¢Ë†Â¼ ΔFEG, Show that:

(i) $\frac{CD}{GH}=\frac{AC}{FG}$

(ii) ΔDCB Ã¢Ë†Â¼ ΔHGE

(iii) ΔDCA Ã¢Ë†Â¼ ΔHGF

Answer :



It is given that ΔABC Ã¢Ë†Â¼ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ ΔACD Ã¢Ë†Â¼ ΔFGH (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Proved above)

∠B = ∠E (Proved above)

∴ ΔDCB Ã¢Ë†Â¼ ΔHGE (By AA similarity criterion)

In ΔDCA and ΔHGF,

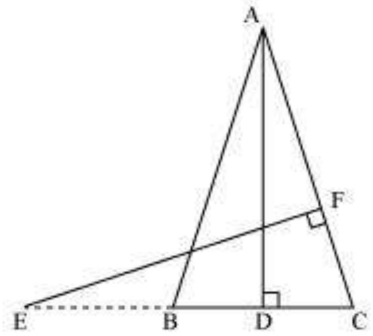
∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ ΔDCA Ã¢Ë†Â¼ ΔHGF (By AA similarity criterion)

Q11 :

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ∝¼ ΔECF



Answer :

It is given that ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ ABD = ∠ ECF

In ΔABD and ΔECF,

∠ ADB = ∠ EFC (Each 90°)

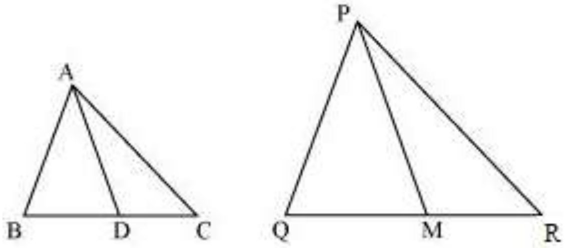
∠ BAD = ∠ CEF (Proved above)

∴ ΔABD ∠Â¼ ΔECF (By using AA similarity criterion)

Q12 :

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ∠Â¼ ΔPQR.

Answer :



Median divides the opposite side.

∴ $BD=\frac{BC}{2}$ and $QM=\frac{QR}{2}$

Given that,

$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{\frac{1}{2}BC}{\frac{1}{2}QR}=\frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$ (Proved above)

∴ ΔABD Ã¢Ë†Â¼ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

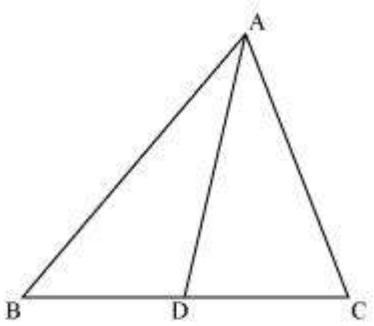
$$\frac{AB}{PQ}=\frac{BC}{QR}$$

∴ ΔABC Ã¢Ë†Â¼ ΔPQR (By SAS similarity criterion)

Q13 :

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that $CA^2 = CB.CD.$

Answer :



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC Ã¢Ë†Â¼ ΔBAC (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

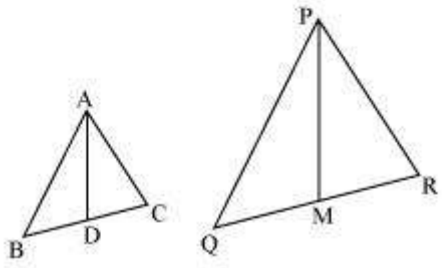
$$\therefore \frac{CA}{CB}=\frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Q14 :

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

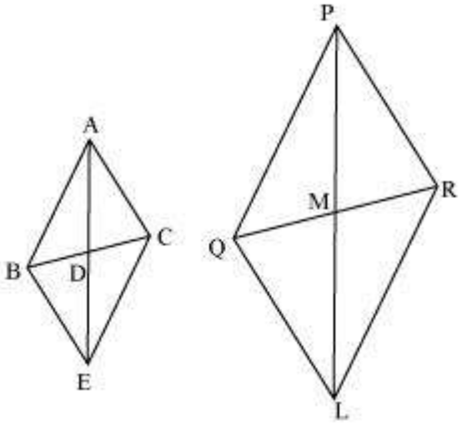
Answer :



Given that,

$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{AE}{PL}$$

∴ ΔABE Ã¢Ë†Â¼ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ ∠BAE = ∠QPL … (1)

Similarly, it can be proved that ΔAEC Ã¢Ë†Â¼ ΔPLR and

∠CAE = ∠RPL … (2)

Adding equation (1) and (2), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ … (3)

In ΔABC and ΔPQR,

$\frac{AB}{PQ} = \frac{AC}{PR}$ (Given)

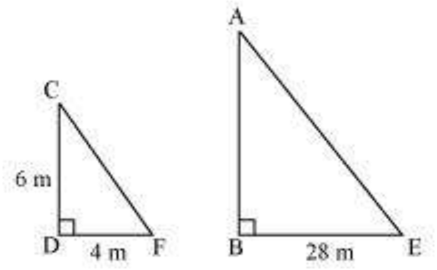
∠CAB = ∠RPQ [Using equation (3)]

∴ ΔABC Ã¢Ë†Â¼ ΔPQR (By SAS similarity criterion)

Q15 :

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer :



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, ∠DCF = ∠BAE

And, ∠DFC = ∠BEA

∠CDF = ∠ABE (Tower and pole are vertical to the ground)

∴ ΔABE Ã¢Ë†Â¼ ΔCDF (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6\text{ m}} = \frac{28}{4}$$

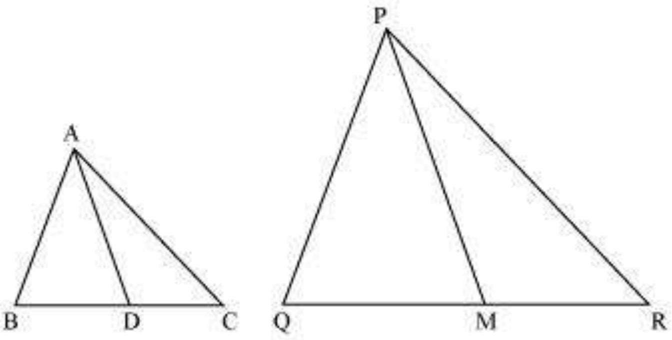
$$\Rightarrow AB = 42\text{ m}$$

Therefore, the height of the tower will be 42 metres.

Q16 :

If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$ prove tha t $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer :



It is given that ΔABC Ã¢Ë†Â¼ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

∴ $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$ … (1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PM are medians, they will divide their opposite sides.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ} = \frac{BD}{QM}$ … (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ} = \frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD Ã¢Ë†Â¼ ΔPQM (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Exercise 6.4 : Solutions of Questions on Page Number : **143**

Q1 :

Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer :

It is given that $\Delta ABC \sim \Delta DEF$.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$EF = 15.4$ cm,

$ar(\Delta ABC) = 64\ \text{cm}^2$,

$ar(\Delta DEF) = 121\ \text{cm}^2$

$$\therefore \frac{ar(ABC)}{ar(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64\ \text{cm}^2}{121\ \text{cm}^2}\right) = \frac{BC^2}{(15.4\ \text{cm})^2}$$

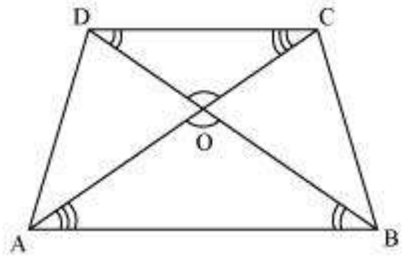
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right)\text{cm}$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right)\text{cm} = (8 \times 1.4)\text{cm} = 11.2\,\text{cm}$$

Q2 :

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer :



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

∴ ΔAOB Ã¢Ë†Â¼ ΔCOD (By AAA similarity criterion)

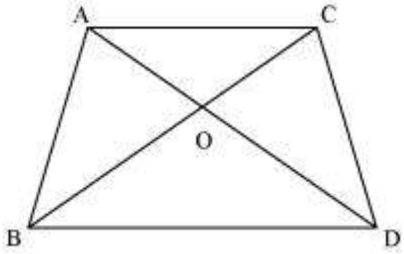
$$\therefore \frac{\text{ar}(\Delta\text{AOB})}{\text{ar}(\Delta\text{COD})} = \left(\frac{\text{AB}}{\text{CD}}\right)^2$$

Since AB = 2 CD,

$$\therefore \frac{\text{ar}(\Delta\text{AOB})}{\text{ar}(\Delta\text{COD})} = \left(\frac{2\text{ CD}}{\text{CD}}\right)^2 = \frac{4}{1} = 4:1$$

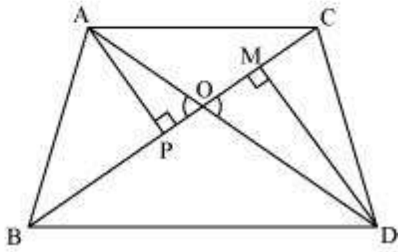
Q3 :

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{area}(\Delta\text{ABC})}{\text{area}(\Delta\text{DBC})} = \frac{\text{AO}}{\text{DO}}$



Answer :

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2}\times \text{Base}\times \text{Height}$

$$\therefore \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)}=\frac{\frac{1}{2}BC\times AP}{\frac{1}{2}BC\times DM}=\frac{AP}{DM}.$$

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO Ã¢Ë†Â¼ ΔDMO (By AA similarity criterion)

$$\therefore \frac{AP}{DM}=\frac{AO}{DO}$$

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DBC)}=\frac{AO}{DO}$$

Q4 :

If the areas of two similar triangles are equal, prove that they are congruent.

Answer :

Let us assume two similar triangles as ΔABC Ã¢Ë†Â¼ ΔPQR.

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta ABC) = $ ar (ΔPQR)

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

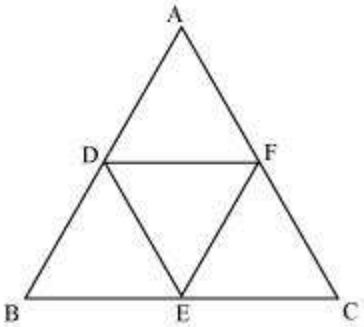
$\Rightarrow AB = PQ, BC = QR,$ and $AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ $\qquad$ (By SSS congruence criterion)

Q5 :

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Answer :



D and E are the mid-points of ΔABC.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2} AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

$\therefore \Delta BED \sim \Delta BCA$ (AAA similarity criterion)

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4} ar(\Delta BCA)$$

Similarly, $ar(\Delta CFE) = \frac{1}{4} ar(CBA)$ and $ar(\Delta ADF) = \frac{1}{4} ar(\Delta ABC)$

Also, $ar(\Delta DEF) = ar(\Delta ABC) - [ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)]$

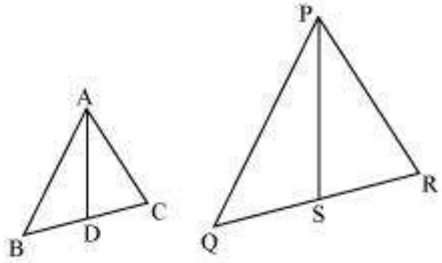
$$\Rightarrow ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4} ar(\Delta ABC) = \frac{1}{4} ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Q6 :

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer :



Let us assume two similar triangles as ΔABC Ã¢Ë†Â¼ ΔPQR. Let AD and PS be the medians of these triangles.

∵ ΔABC Ã¢Ë†Â¼ ΔPQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ …(1)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PS are medians,

∴ BD = DC = $\frac{BC}{2}$

And, QS = SR = $\frac{QR}{2}$

Equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ … (3)

In ΔABD and ΔPQS,

∠B = ∠Q [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

∴ ΔABD Ã¢Ë†Â¼ ΔPQS (SAS similarity criterion)

Therefore, it can be said that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$ … (4)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

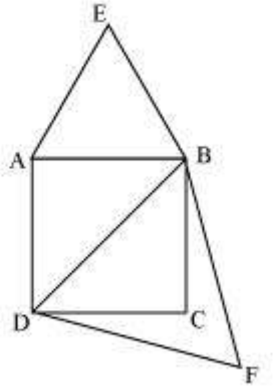
And hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Q7 :

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer :



Let ABCD be a square of side a.

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60 º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of }\Delta\,\text{ABE}}{\text{Area of }\Delta\,\text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Q8 :

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

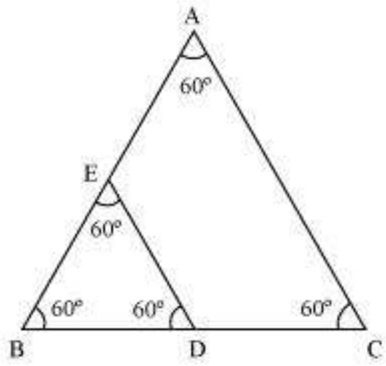
(A) 2 : 1

(B) 1 : 2

(C) 4 : 1

(D) 1 : 4

Answer :



We know that equilateral triangles have all its angles as 60 º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = *x*

Therefore, side of $\Delta\text{BDE} = \frac{x}{2}$

$$\therefore \quad \frac{\text{area}(\Delta\,\text{ABC})}{\text{area}(\Delta\,\text{BDE})} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Q9 :

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Answer :

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Exercise 6.5 : Solutions of Questions on Page Number : **150**

Q1 :

Sides of triangles are given below. Determine which of them are right triangles? In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer :

(i) It is given that the sides of the triangle are 7 cm, 24 cm, and 25 cm.

Squaring the lengths of these sides, we will obtain 49, 576, and 625.

49 + 576 = 625

Or, $7^2 + 24^2 = 25^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 25 cm.

(ii) It is given that the sides of the triangle are 3 cm, 8 cm, and 6 cm.

Squaring the lengths of these sides, we will obtain 9, 64, and 36.

However, 9 + 36 ≠ 64

Or, $3^2 + 6^2 \neq 8^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iii)Given that sides are 50 cm, 80 cm, and 100 cm.

Squaring the lengths of these sides, we will obtain 2500, 6400, and 10000.

However, 2500 + 6400 ≠ 10000

Or, $50^2 + 80^2 \neq 100^2$

Clearly, the sum of the squares of the lengths of two sides is not equal to the square of the length of the third side.

Therefore, the given triangle is not satisfying Pythagoras theorem.

Hence, it is not a right triangle.

(iv)Given that sides are 13 cm, 12 cm, and 5 cm.

Squaring the lengths of these sides, we will obtain 169, 144, and 25.

Clearly, 144 +25 = 169

Or, $12^2 + 5^2 = 13^2$

The sides of the given triangle are satisfying Pythagoras theorem.

Therefore, it is a right triangle.

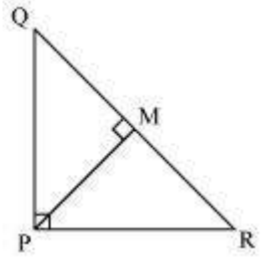
We know that the longest side of a right triangle is the hypotenuse.

Therefore, the length of the hypotenuse of this triangle is 13 cm.

Q2 :

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that PM^2 = QM x MR.

Answer :

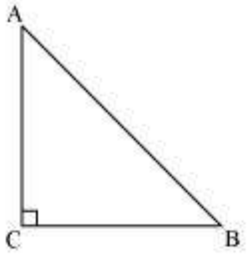


$$\text{Let } \angle MPR = x$$
$$\text{In} \Delta MPR,$$
$$\angle MRP = 180^\circ - 90^\circ - x$$
$$\angle MRP = 90^\circ - x$$
$$\text{Similarly, in} \Delta MPQ,$$
$$\angle MPQ = 90^\circ - \angle MPR$$
$$= 90^\circ - x$$
$$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x)$$
$$\angle MQP = x$$
$$\text{In} \Delta\ QMP \text{ and } \Delta PMR,$$
$$\angle MPQ = \angle MRP$$
$$\angle PMQ = \angle RMP$$
$$\angle MQP = \angle MPR$$
$$\therefore \Delta QMP \sim \Delta PMR \qquad (\text{By AAA similarity criterion})$$
$$\Rightarrow \frac{QM}{PM} = \frac{MP}{MR}$$
$$\Rightarrow PM^2 = QM \times MR$$

Q3 :

ABC is an isosceles triangle right angled at C. prove that $AB^2 = 2\ AC^2$.

Answer :



Given that ΔABC is an isosceles triangle.

∴ AC = CB

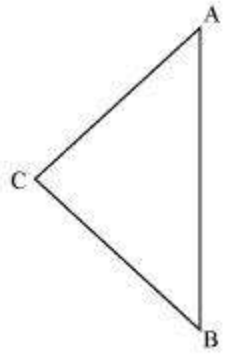
Applying Pythagoras theorem in ΔABC (i.e., right-angled at point C), we obtain

$$AC^2 + CB^2 = AB^2$$
$$\Rightarrow AC^2 + AC^2 = AB^2 \qquad (AC = CB)$$
$$\Rightarrow 2AC^2 = AB^2$$

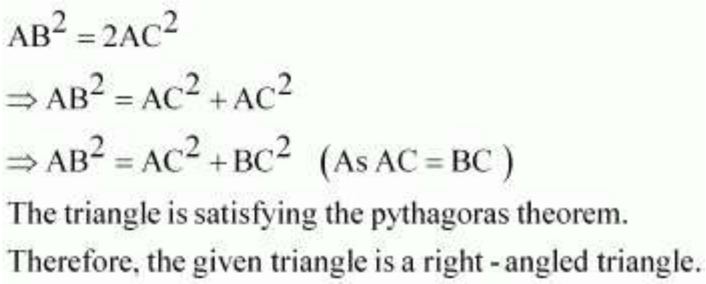
Q4 :

ABC is an isosceles triangle with AC = BC. If $AB^2 = 2\ AC^2$, prove that ABC is a right triangle.

Answer :



Given that,



$$AB^2 = 2AC^2$$

$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \quad (\text{As } AC = BC)$$

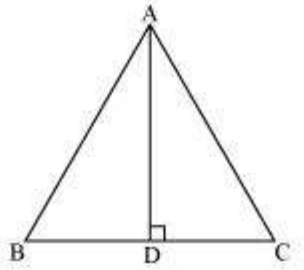
The triangle is satisfying the pythagoras theorem.

Therefore, the given triangle is a right - angled triangle.

Q5 :

ABC is an equilateral triangle of side 2*a*. Find each of its altitudes.

Answer :



Let AD be the altitude in the given equilateral triangle, ΔABC.

We know that altitude bisects the opposite side.

∴ BD = DC = *a*

In ΔADB,

$\angle ADB = 90°$

Applying pythagoras theorem, we obtain

$$AD^2 + DB^2 = AB^2$$
$$\Rightarrow AD^2 + a^2 = (2a)^2$$
$$\Rightarrow AD^2 + a^2 = 4a^2$$
$$\Rightarrow AD^2 = 3a^2$$
$$\Rightarrow AD = a\sqrt{3}$$

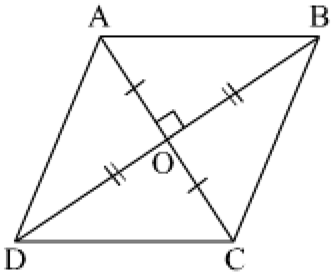
In an equilateral triangle, all the altitudes are equal in length.

Therefore, the length of each altitude will be $\sqrt{3}a$.

Q6 :

Prove that the sum of the squares of the sides of rhombus is equal to the sum of the squares of its diagonals.

Answer :



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \qquad \ldots (1)$$

$$BC^2 = BO^2 + OC^2 \qquad \ldots (2)$$

$$CD^2 = CO^2 + OD^2 \qquad \ldots (3)$$

$$AD^2 = AO^2 + OD^2 \qquad \ldots (4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

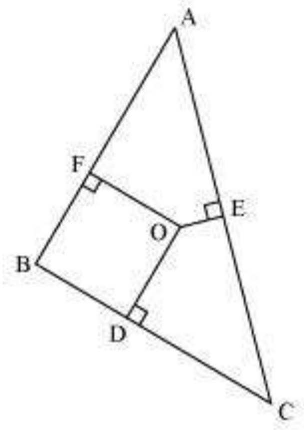
(Diagonals bisect each other)

$$=2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

$$= (AC)^2 + (BD)^2$$

Q7 :

In the following figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

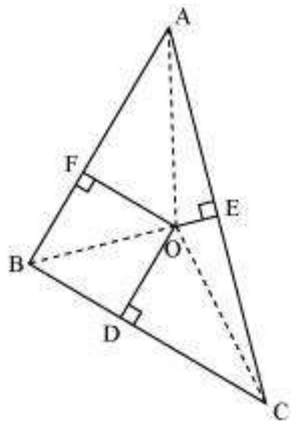


(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer :

Join OA, OB, and OC.



(i) Applying Pythagoras theorem in ΔAOF, we obtain

$$OA^2 = OF^2 + AF^2$$

Similarly, in ΔBOD,

$$OB^2 = OD^2 + BD^2$$

Similarly, in ΔCOE,

$$OC^2 = OE^2 + EC^2$$

Adding these equations,

$$OA^2 + OB^2 + OC^2 = OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$$

$$OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + EC^2$$

(ii) From the above result,

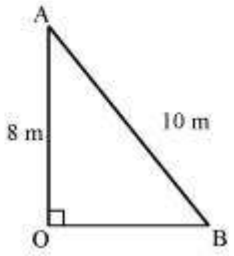
$$AF^2 + BD^2 + EC^2 = \left(OA^2 - OE^2\right) + \left(OC^2 - OD^2\right) + \left(OB^2 - OF^2\right)$$

$$\therefore AF^2 + BD^2 + EC^2 = AE^2 + CD^2 + BF^2$$

Q8 :

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

Answer :



Let OA be the wall and AB be the ladder.

Therefore, by Pythagoras theorem,

$$
\begin{aligned}
&AB^2 = OA^2 + BO^2 \\
&(10\text{ m})^2 = (8\text{ m})^2 + OB^2 \\
&100\text{ m}^2 = 64\text{ m}^2 + OB^2 \\
&OB^2 = 36\text{ m}^2 \\
&OB = 6\text{ m}
\end{aligned}
$$

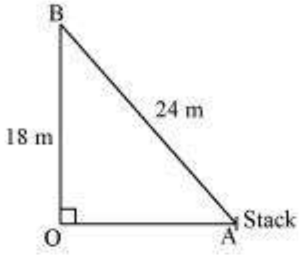
Therefore, the distance of the foot of the ladder from the base of the wall is

6 m.

Q9 :

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

Answer :



Let OB be the pole and AB be the wire.

By Pythagoras theorem,

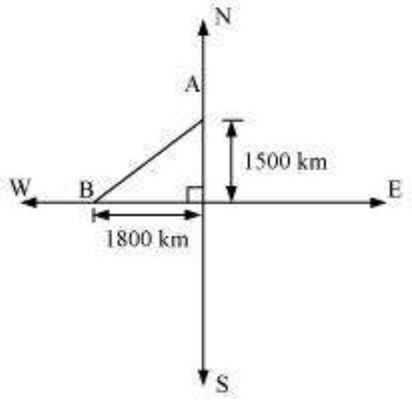
$$
\begin{aligned}
&AB^2 = OB^2 + OA^2 \\
&(24\text{ m})^2 = (18\text{ m})^2 + OA^2 \\
&OA^2 = (576 - 324)\text{m}^2 = 252\text{ m}^2 \\
&OA = \sqrt{252}\text{ m} = \sqrt{6\times6\times7}\text{ m} = 6\sqrt{7}\text{ m}
\end{aligned}
$$

Therefore, the distance from the base is $6\sqrt{7}$ m.

Q10 :

An aeroplane leaves an airport and flies due north at a speed of 1,000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1,200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer :



Distance travelled by the plane flying towards north in $1\frac{1}{2}\text{hrs}$ $=1,000\times1\frac{1}{2}$ $=1,500\text{ km}$

Similarly, distance travelled by the plane flying towards west in $1\frac{1}{2}\text{hrs}$ $=1,200\times1\frac{1}{2}=1,800\text{ km}$

Let these distances be represented by OA and OB respectively.

Applying Pythagoras theorem,

Distance between these planes after $1\frac{1}{2}\text{hrs}$, AB = $\sqrt{\text{OA}^2 + \text{OB}^2}$

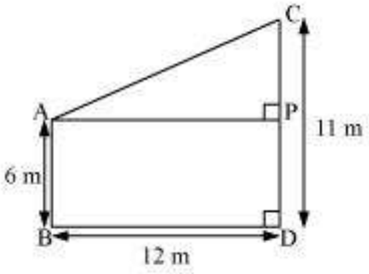
$$=\left(\sqrt{(1,500)^2+(1,800)^2}\right)\text{km}=\left(\sqrt{2250000+3240000}\right)\text{km}$$
$$=\left(\sqrt{5490000}\right)\text{km}=\left(\sqrt{9\times610000}\right)\text{km}=300\sqrt{61}\text{ km}$$

Therefore, the distance between these planes will be $300\sqrt{61}$ km after $1\frac{1}{2}\text{hrs}$.

Q11 :

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer :



Let CD and AB be the poles of height 11 m and 6 m.

Therefore, CP = 11 - 6 = 5 m

From the figure, it can be observed that AP = 12m

Applying Pythagoras theorem for ΔAPC, we obtain

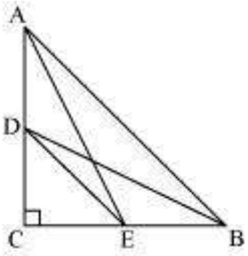
$$
\begin{aligned}
&\mathrm{AP}^2+\mathrm{PC}^2=\mathrm{AC}^2\\
&(12\text{ m})^2+(5\text{ m})^2=\mathrm{AC}^2\\
&\mathrm{AC}^2=(144+25)\mathrm{m}^2=169\text{ m}^2\\
&\mathrm{AC}=13\text{ m}
\end{aligned}
$$

Therefore, the distance between their tops is 13 m.

Q12 :

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that AE^2+ BD^2 = AB^2 + DE^2

Answer :



Applying Pythagoras theorem in ΔACE, we obtain

$$AC^2 + CE^2 = AE^2 \quad \ldots (1)$$

Applying Pythagoras theorem in ΔBCD, we obtain

$$BC^2 + CD^2 = BD^2 \quad \ldots (2)$$

Using equation (1) and equation (2), we obtain

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + BD^2 \quad \ldots (3)$$

Applying Pythagoras theorem in ΔCDE, we obtain

$$DE^2 = CD^2 + CE^2$$

Applying Pythagoras theorem in ΔABC, we obtain

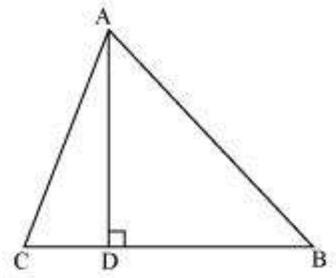
$$AB^2 = AC^2 + CB^2$$

Putting the values in equation (3), we obtain

$$DE^2 + AB^2 = AE^2 + BD^2$$

Q13 :

The perpendicular from A on side BC of a ΔABC intersect BC at D such that DB = 3 CD. Prove that 2 AB^2 = 2 AC^2 + BC^2



Answer :

Applying Pythagoras theorem for ΔACD, we obtain

$$AC^2 = AD^2 + DC^2$$

$$AD^2 = AC^2 - DC^2 \quad \ldots (1)$$

Applying Pythagoras theorem in ΔABD, we obtain

$AB^2 = AD^2 + DB^2$

$AD^2 = AB^2 - DB^2 \qquad \ldots (2)$

From equation (1) and equation (2), we obtain

$AC^2 - DC^2 = AB^2 - DB^2 \qquad \ldots (3)$

It is given that 3DC = DB

$\therefore DC = \frac{BC}{4}$ and $DB = \frac{3BC}{4}$

Putting these values in equation (3), we obtain

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

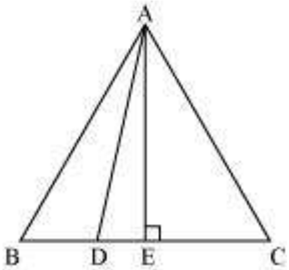
$$16AB^2 - 16AC^2 = 8BC^2$$

$$2AB^2 = 2AC^2 + BC^2$$

Q14 :

In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$ BC. Prove that 9 AD^2 = 7 AB^2.

Answer :



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

$\therefore$ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

And, AE = $\frac{a\sqrt{3}}{2}$

Given that, BD = $\frac{1}{3}$ BC

∴ BD = $\frac{a}{3}$

DE = BE - BD = $\frac{a}{2} - \frac{a}{3} = \frac{a}{6}$

Applying Pythagoras theorem in ΔADE, we obtain

$AD^2 = AE^2 + DE^2$

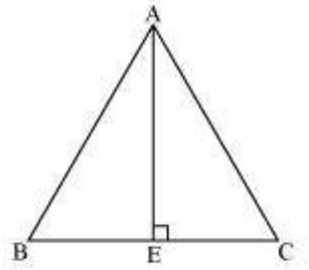
$$\begin{aligned} AD^2 &= \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2 \\ &= \left(\frac{3a^2}{4}\right) + \left(\frac{a^2}{36}\right) \\ &= \frac{28a^2}{36} \\ &= \frac{7}{9}AB^2 \end{aligned}$$

⇒ $9\ AD^2 = 7\ AB^2$

Q15 :

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer :



Let the side of the equilateral triangle be *a*, and AE be the altitude of ΔABC.

∴ BE = EC = $\frac{BC}{2} = \frac{a}{2}$

Applying Pythagoras theorem in ΔABE, we obtain

$AB^2 = AE^2 + BE^2$

$$a^2 = AE^2 + \left(\frac{a}{2}\right)^2$$

$$AE^2 = a^2 - \frac{a^2}{4}$$

$$AE^2 = \frac{3a^2}{4}$$

$4AE^2 = 3a^2$

⇒ 4 × (Square of altitude) = 3 × (Square of one side)

Q16 :

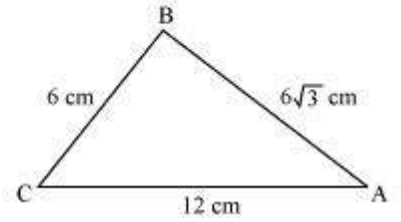
Tick the correct answer and justify: In ΔABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

The angle B is:

(A) 120° (B) 60°

(C) 90° (D) 45°

Answer :



Given that, AB = $6\sqrt{3}$ cm, AC = 12 cm, and BC = 6 cm

It can be observed that

$AB^2 = 108$

$AC^2 = 144$

And, $BC^2 = 36$

$AB^2 + BC^2 = AC^2$

The given triangle, ΔABC, is satisfying Pythagoras theorem.

Therefore, the triangle is a right triangle, right-angled at B.

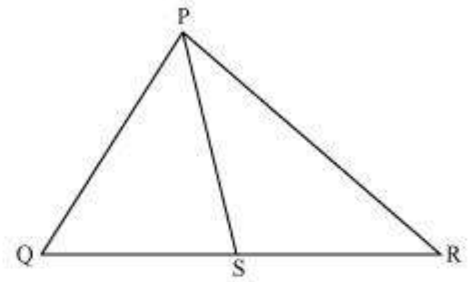
∴ ∠B = 90°

Hence, the correct answer is (C).

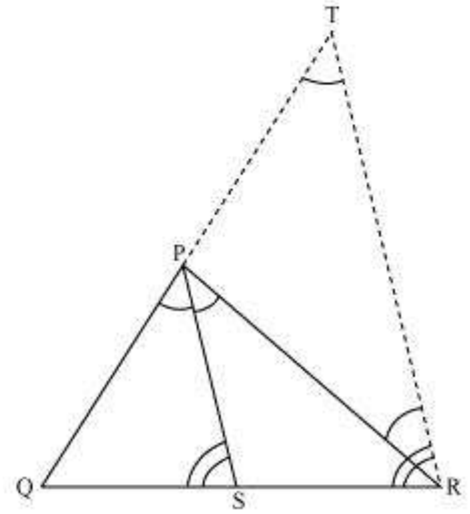
Exercise 6.6 : Solutions of Questions on Page Number : **152**

Q1 :

In the given figure, PS is the bisector of ∠QPR of ΔPQR. Prove that $\frac{QS}{SR}=\frac{PQ}{PR}$ **.**



Answer :



Let us draw a line segment RT parallel to SP which intersects extended line segment QP at point T.

Given that, PS is the angle bisector of ∠QPR.

∠QPS = ∠SPR … (1)

By construction,

∠SPR = ∠PRT (As PS || TR) … (2)

∠QPS = ∠QTR (As PS || TR) … (3)

Using these equations, we obtain

∠PRT = ∠QTR

∴ PT = PR

By construction,

PS || TR

By using basic proportionality theorem for ΔQTR,

QSSR=QPPT

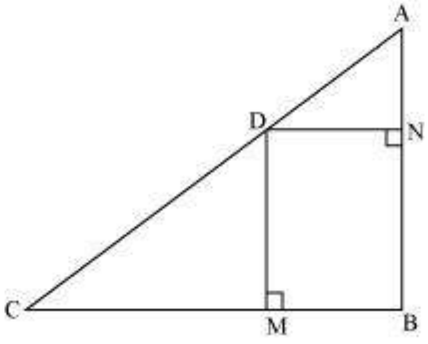
⇒QSSR

Q2 :

In the given figure, D is a point on hypotenuse AC of ΔABC, DM ⊥ BC and DN ⊥ AB, Prove that:

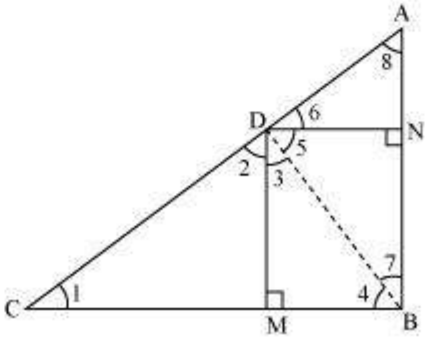
(i) DM^2 = DN.MC

(ii) DN^2 = DM.AN



Answer :

(i)Let us join DB.



We have, DN || CB, DM || AB, and ∠B = 90°

∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

The condition to be proved is the case when D is the foot of the perpendicular drawn from B to AC.

∴ ∠CDB = 90°

⇒ ∠2 + ∠3 = 90° … (1)

In ΔCDM,

∠1 + ∠2 + ∠DMC = 180°

⇒ ∠1 + ∠2 = 90° … (2)

In ΔDMB,

∠3 + ∠DMB + ∠4 = 180°

⇒ ∠3 + ∠4 = 90° … (3)

From equation (1) and (2), we obtain

∠1 = ∠3

From equation (1) and (3), we obtain

∠2 = ∠4

In ΔDCM and ΔBDM,

∠1 = ∠3 (Proved above)

∠2 = ∠4 (Proved above)

∴ ΔDCM Ã¢Ë†Â¼ ΔBDM (AA similarity criterion)

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC}$$

$$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \qquad (BM = DN)$$

⇒ $DM^2 = DN \times MC$

(ii) In right triangle DBN,

∠5 + ∠7 = 90° … (4)

In right triangle DAN,

∠6 + ∠8 = 90° … (5)

D is the foot of the perpendicular drawn from B to AC.

∴ ∠ADB = 90°

⇒ ∠5 + ∠6 = 90° … (6)

From equation (4) and (6), we obtain

∠6 = ∠7

From equation (5) and (6), we obtain

∠8 = ∠5

In ΔDNA and ΔBND,

∠6 = ∠7 (Proved above)

∠8 = ∠5 (Proved above)

∴ ΔDNA Ã¢Ë†Â¼ ΔBND (AA similarity criterion)

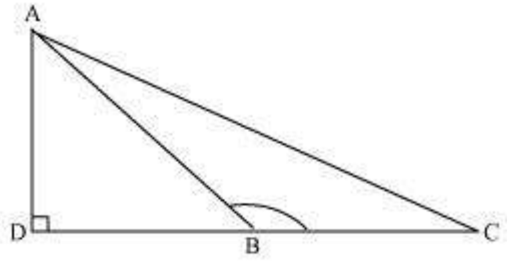
$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB}$$

⇒ $DN^2 = AN \times NB$

⇒ $DN^2 = AN \times DM$ (As NB = DM)

Q3 :

In the given figure, ABC is a triangle in which ∠ ABC> 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer :

Applying Pythagoras theorem in ΔADB, we obtain

$AB^2 = AD^2 + DB^2$ … (1)

Applying Pythagoras theorem in ΔACD, we obtain

$AC^2 = AD^2 + DC^2$

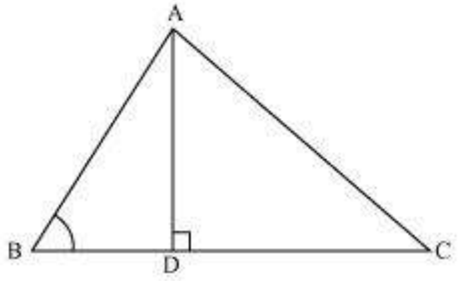
$AC^2 = AD^2 + (DB + BC)^2$

$AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$AC^2 = AB^2 + BC^2 + 2DB \times BC$ [Using equation (1)]

Q4 :

In the given figure, ABC is a triangle in which ∠ ABC < 90° and AD ⊥ BC. Prove that $AC^2 = AB^2 + BC^2 - 2BC.BD$.



Answer :

Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$

⇒ $AD^2 = AB^2 - DB^2$ … (1)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 - BD^2 + DC^2 = AC^2$ [Using equation (1)]

$AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

$= AB^2 + BC^2 - 2BC \times BD$

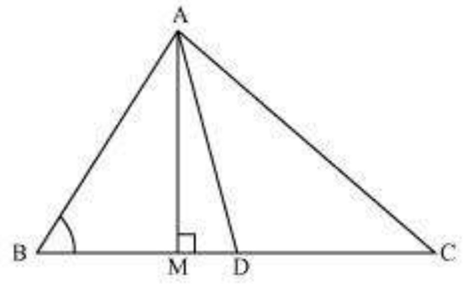
Q5 :

In the given figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:

(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(iii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Answer :

(i) Applying Pythagoras theorem in ΔAMD, we obtain

$AM^2 + MD^2 = AD^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$

$AM^2 + (MD + DC)^2 = AC^2$

$(AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$AD^2 + DC^2 + 2MD.DC = AC^2$ [Using equation (1)]

Using the result, $DC = \frac{BC}{2}$, we obtain

$$AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2$$

$$AD^2 + \left(\frac{BC}{2}\right)^2 + MD \times BC = AC^2$$

(ii) Applying Pythagoras theorem in ΔABM, we obtain

$AB^2 = AM^2 + MB^2$

$= (AD^2 - DM^2) + MB^2$

$= (AD^2 - DM^2) + (BD - MD)^2$

$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$

$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right) \times MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - BC \times MD$$

(iii)Applying Pythagoras theorem in ΔABM, we obtain

$AM^2 + MB^2 = AB^2$ … (1)

Applying Pythagoras theorem in ΔAMC, we obtain

$AM^2 + MC^2 = AC^2$ … (2)

Adding equations (1) and (2), we obtain

$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$

$2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$

$2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$

$2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD (- BD + DC) = AB^2 + AC^2$

$$2\left(AM^2 + MD^2\right) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

$$2AD^2 + \frac{BC^2}{2} = AB^2 + AC^2$$